JEE MAIN - 2017 / [SET-A] / SOLUTIONS
PHYSICS

DISHA CLASSES Test code
PHYSICS. → [A]

1. (1) Initial stress $= \frac{mg}{A}$

Final stress $= \frac{(9\times9\times9)mg}{9\times9A}$

[volume increases 9×9×9 times & area of foot increases by 9×9 times]

So new stress becomes 9 times

2. (3) acceleration is uniform so v-t graph will be straight line
take upward dir^n +ve & downwards -ve
velocity is zero at the top most pt.

3. (3) $m\frac{dv}{dt} = -Kv^2 \Rightarrow m\frac{dv}{v^2} = -K\,dt$

$\Rightarrow m\left[\frac{1}{v}\right]_{v_0}^{v} = K[t]_0^{10}$

as KE i{n}itial is $\frac{1}{4}$th then $v = \frac{v_0}{2}$

So $K = 10^{-4}\ kg\ m^{-1}$

4. (1) $F = 6t$

$m\frac{dv}{dt} = 6t \Rightarrow m\int_0^v dv = 6\int_0^1 t\,dt$

$mv = 3\times1 \Rightarrow v = 3$

KE final $= \frac{1}{2}\times1\times9 = 4.5\ J$.
Initial Kinetic energy $= 0\ J$.
So work done $= 4.5\ J$

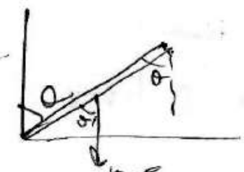
5. (1) $I = \frac{1}{4}mR^2 + \frac{1}{12}ml^2 = \frac{m}{4}\left(R^2 + \frac{l^2}{3}\right)$

$m = \pi R^2 \rho l \Rightarrow R^2 = \frac{m}{\pi \rho l}$

$I = \frac{m}{4}\left(\frac{m}{\pi \rho l} + \frac{l^3}{3}\right)$

Keep $\frac{dI}{dl} = 0$ & get $\frac{l}{R} = \sqrt{\frac{3}{2}}$

6. (1)



$mg\frac{l}{2}\sin\theta = I\alpha$

$mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2 \times \alpha$

$\Rightarrow \alpha = \frac{3}{2}\frac{g\sin\theta}{l}$

7. (4) g inside the earth is $\frac{GMx}{R^3}$

g outside the earth is $\frac{GM}{x^2}$

8. (2) Heat given by the copper ball = Heat taken by the water + Heat taken by calorimeter

$100 \times 0.1 \times (T-75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$\Rightarrow T = 885$

9. (1) decrease of volume due to pressure = increase in volume due to temp.

$\frac{P}{K}V = V \times 3\alpha\Delta t$ $\left[K = \frac{P}{\Delta V/V}\right]$

$\Rightarrow \Delta t = \frac{P}{3\alpha K}$

10. (3) $C_p - C_v = R$ ∵ [C_p & C_v are molar specific heats]

In the question C_p & C_v are given specific heats

$C_p - C_v$ is same for all ideal gases if they are not of same atomicity.

So $a = \frac{R}{2}$ & $b = \frac{R}{28}$

Q.11 (4)

$$n_f = \frac{10^5 \times 10^{-3}}{R\ 300} \times N_0 \ \& \ n_i = \frac{10^5 \times 10^{-3}}{R \times 290} \times N_0$$

Q.12. (4) Kinetic energy is max. at the mean position & 0 after time $\frac{T}{4}$

13. (3) $f' = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\, f$

$v = \frac{c}{2}$

Q.14 (3) 

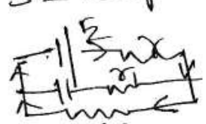
$\tau_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E\hat{i}$ & $\vec{\tau}_1 = -\vec{\tau}_2$

$\tau_2 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times \{E\,\hat{i} + \sqrt{3}E\,\hat{j}\}$ So $\theta = 60°$

15. (4) four capacitor will be connected in one row, so the capacitance of one row $= (\frac{1}{4}\mu f)$ & there will be 8 rows.

So net capacitance $= 2\mu f$

So 32 capacitors.

16. (3)  The current will flow only in ~~middle~~ r_2 outer circuit. The voltage across $r_2 = \left(\frac{E r_2}{r+r_2}\right) \rightarrow$ Same will be the voltage across capacitor. So the charge $q = \frac{CE r_2}{r_2 + r_2}$

17. (4) Net EMF across each loop is zero and across whole loop is also zero.

18. (1) $\tau = MB\sin\theta = I\alpha$

$MB\theta = I\alpha$

$T = .665\,sec$ $\quad T = 2\pi\sqrt{\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{I}{MB}}$

19. (1) $V = i(R_G + R_H)$

$10 = 5\times10^{-3}(15+R_H)$

$\Rightarrow R_H = 1985\,\Omega$.

20. (3)

$\Delta q = \frac{\Delta\phi}{R} \Rightarrow \Delta\phi = \Delta q \times R = \frac{1}{2}\times10\times0.5\times100$

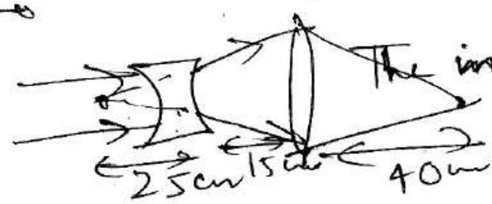
Δq (charge flow) = Area b/w i & t

21. (1) $\frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} = \frac{hc}{eV}$

$\ln\lambda_{min} = \ln\left(\frac{hc}{e}\right) - \ln V$

~~So~~

22. (1)



The image made by concave lens will be 25 cm behind & so it is at the center of curvature of the convex lens.

23. (2) $\frac{Dn_1\lambda_1}{d} = \frac{Dn_2\lambda_2}{d}$

$n_1\times520 = n_2\times650 \Rightarrow n_1 = 5$

$x = \frac{Dn\lambda}{d} = \frac{1.5\times5\times520\times10^{-9}}{0.5\times10^{-3}}$

24. (2) B.C: $(m)\to v$ $\quad(\frac{m}{2})\to 0$ | A.C: $(m)\to v_1$ $\quad(\frac{m}{2})\to v_2$

$mv = mv_1 + \frac{m}{2}v_2$

$1 = -\frac{(v_2-v_1)}{0-v}$

$v_1 = \frac{v}{3}$, $v_2 = \frac{4v}{3}$

$\lambda_A = \frac{h3}{mv}$

$\lambda_B = \frac{h\times3}{4vm}$

25. (4) $\frac{hc}{\lambda_2} = \{-E - (-\frac{4}{3}E)\} = \frac{E}{3}$

$\frac{hc}{\lambda_1} = \{-E - (-2E)\} = E$

So $\frac{\lambda_1}{\lambda_2} = \frac{1}{3}$

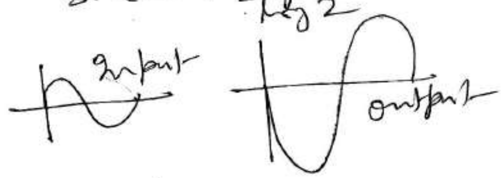
26. (2) $N = N_0 e^{-\lambda t}$

$\frac{N_B}{N_A} = \frac{N_0 - N}{N} = 0.3$ so $\frac{N_0 - N_0 e^{-\lambda t}}{N_0 e^{-\lambda t}} = 0.3$

$\Rightarrow 1 = 1.3 e^{-\lambda t}$

So $t = T\frac{\log 1.3}{\log 2}$

27. (4)



28. (1)

29. (2)

30. (2) $T = \frac{D}{2}\frac{h}{2} \times g \times 10^{-3}$

$\frac{dT}{T} = \frac{d(D)}{D} + \frac{dh}{h}$ [g is not a measured parameter]

CHEMISTRY

MAT-42 PART-B chemistry

(31) Given

$C + O_2 \rightarrow CO_2 \quad \Delta H_1 = -393.5$ kJ/mol — (I)

$H_2 + \frac{1}{2}O_2 \rightarrow H_2O \quad \Delta H_2 = -285.8$ kJ/mol — (II)

$CO_2 + 2H_2O \rightarrow CH_4 + 2O_2 \rightarrow \Delta H_3 = 890.3$ kJ/mol — (III)

$C + 2H_2 \longrightarrow CH_4 — \Delta H_4 = ?$ — (4)

if eq. (I) + (II)×2 + (III) = eq. (4)

so $\Delta H_1 + 2\times\Delta H_2 + \Delta H_3 = \Delta H_4$

$-393.5 + 2\times(-285.8) + 890.3 = \Delta H_4$

$\boxed{\Delta H_4 = -74.8 \text{ kJ/mol}}$ Ans

(32) $M_2CO_3 + 2HCl \longrightarrow MCl + H_2O + CO_2$

Unitry method

1 mole of M_2CO_3 Produce = 1 mole of CO_2

so .01186 mole of CO_2 Produce by = .01186 mole M_2CO_3

So mole of M_2CO_3 = .01186

$$\frac{w}{M} = .01186$$

$$\frac{1}{M} = .01186$$

$$\boxed{M = \frac{1}{.01186} = 84.3 \text{ gm/mol}}$$

(33) option-1

in adiabatic $\Delta H = 0$

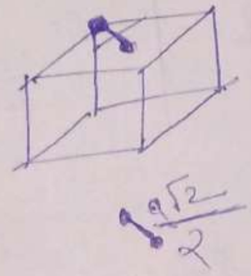
so $\Delta U = \Delta H + P\Delta V$

$\Delta U = 0 + P\Delta V$

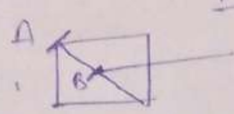
$\boxed{\Delta U = 14}$

(34) → option → (4)

(35) → option → 2



in fcc → $\boxed{r = \frac{a}{\sqrt{2}}}$



$BB = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

(36) lower the value of Reduction potential higher its oxidation nature so it higher its reducing agent.

option - 3

(37) option → (2)

$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$

$1-\alpha \qquad \alpha/2$

we know that

$\Delta T_f = i \cdot k_b \times m$

$.45 = i \times 5.12 \times \frac{.2 \times 1000}{60 \times 20}$

$i = .527$

& $i = 1 - \frac{\alpha}{2}$ | So $\alpha = .946$

$\boxed{\alpha = 94.6\%}$

(38) option → (2)

we know that Bohr radius equation

$r = .529\,\mathring{A} \times \frac{n^2}{Z}$

$r = .529 \times \frac{2^2}{1}$

$= 2.21\,\mathring{A}$

(39) option → (2)

$k = A e^{-E_a/RT}$

$\ln \frac{k_2}{k_1} = \frac{E_{a_1} - E_{a_2}}{RT}$

$= \frac{10 \times 1000}{300 \times 8.3} = 4$

(40) for weak & strong electrolyte

$$pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{3.2}{2} - \frac{3.4}{2}$$

$$\boxed{pH = 6.9} \rightarrow \text{option} \rightarrow 4$$

(41) option. 3

(42) Option → (4)

acc. to MOT → $CO = 14e^-$

$\sigma 1s \quad \overset{*}{\sigma} 1s \quad \sigma 2s \quad \overset{*}{\sigma} 2s \quad \pi 2p_x = \pi 2p_y \quad \sigma 2p_z \quad \overset{*}{\pi} 2p_x = \overset{*}{\pi} 2p_y$

$2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$

So all paired so CO is not paramagnetic

(43) option → (3)

$$\overset{+4}{XeF_4} + O_2F_2 \rightarrow \overset{+6}{XeF_6}$$

(44) we know ~~that~~ (option → 1)

level of F < 1 ppm, SO_4^{2-} < 500 ppm

& NO_3^- = 50 ppm this data given suitable

for drinking so in option the value

of F^- is more then ~~given~~ refrence data so 1 is right

(45) option → $\underline{\underline{3}}$

isoelectronic → same no. of e^-

O^{2-} F^- Na^+ Mg^{2+}

$e^- \Rightarrow$ 10 10 10 10

(46) → option → $\underline{1}$

$Cl_2 + 2NaOH \rightarrow NaCl + NaOCl + H_2O$

Cl^- OCl^-

(47) option → (2)

ZnO is Amphoteric in nature

in case → $\underline{\underline{I}}$ Na_2O is base so ZnO is Acid

case → $\underline{II}$ CO_2 is acid so Na_2O is Base

(48) option → 2

$Na_2C_2O_4 + CaCl_2 \rightarrow CaC_2O_4 \downarrow$

white ppt

(49) option → 1

wt. of Hydrogen $= 75 \times \frac{10}{100} = 7.5$ kg

if we replace ^1H to ^2H

then new weight $= 7.5 \times 2 = 15$ kg

So weight increases $= 15 - 7.5$

$= \boxed{7.5 \text{ kg}}$

(50) option → (2)

$CoCl_3 \cdot 6H_2O + AgNO_3 \rightarrow$ product
$AgCl$ + any product

no. of AgCl produced $= \frac{1.2 \times 10^{22}}{6 \times 10^{23}}$

$= .02$ mole

of mole of complex $= \frac{100 \times .1 \times}{1000}$

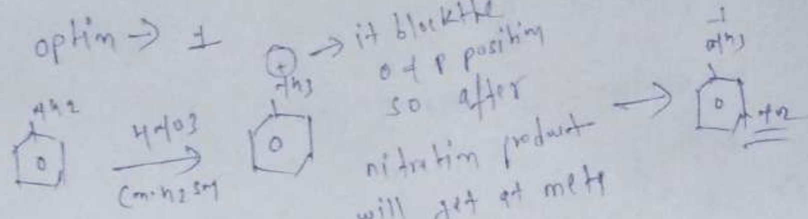
$= .01$

so .01 mole complex gives .02 AgCl

means 2 Cl^- ionised

So formula $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

(51) option → 1



$\oplus$ → it blocks the o & p position so after nitration product will get at meta

(52) option → 3
i think in 3rd option there is no β-H to go elimination so my be (3) but check it.

(53) option → 3
nylon-6

(54) option → 2
there is no resonance

(55) option → 4
III is stablized by resonace
1 is 2° carbocation

(56) option → 4
it will give elimination reaction

(57) → (3) option
$\boxed{\text{presence of easter}}$

(58) option → (2)
$CH_3-CH=\underset{CH_3}{\underset{|}{C}}-CH_2-CH_3$
$\downarrow$ HBr, per.
$CH_3-CH-CH-C-C$
$\quad\quad\quad\quad |$
$\quad\quad\quad CH_3$
2 chiral so stereo $= 2^2 = 4$

(59) – (3) grignard

(60) – (4) DIBAL → it reduces into CHO

MAHEMATICS

(61) let $y = \frac{x}{1+x^2} \Rightarrow y + yx^2 = x$

$\Rightarrow yx^2 - x + y = 0$

Since x is real $\Rightarrow D \geq 0 \Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow (-1)^2 - 4 \cdot y \cdot y \geq 0$

$\Rightarrow 1 - 4y^2 \geq 0$

$\Rightarrow \frac{1}{4} \geq y^2$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$ so range $= [-\frac{1}{2}, \frac{1}{2}]$

is equal to codomain $\Rightarrow$ into function

$$f'(x) = \frac{(1+x^2)\times 1 - x \times 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

Since $1-x^2$ is +ive as well as −ive $\Rightarrow$

$f'(x)$ can take both +ive & −ive values

$\Rightarrow f(x)$ is increasing as well as decreasing

$\Rightarrow$ many one function

So (2) is correct.

(62) Eqn can be written as $\sum_{r=1}^{n} (x+r-1)(x+r) = 10n$

$$\Rightarrow \sum_{r=1}^{n} \left[x^2 + (2r-1)x + r^2 - r\right] = 10n$$

$$\Rightarrow nx^2 + x\sum_{r=1}^{n}(2r-1) + \frac{1}{6}n(n+1)(2n+1) - \frac{n(n+1)}{2} = 10n$$

$$\Rightarrow nx^2 + x(n^2) + \frac{1}{6}n(n+1)(2n+1) - \frac{n(n+1)}{2} = 10n$$

Cancelling n from both sides

$$\Rightarrow x^2 + nx + \frac{(n+1)(2n+1)}{6} - \frac{(n+1)}{2} = 10$$

$$\Rightarrow x^2 + nx + \left(\frac{n+1}{6}\right)[2n+1-3] = 10$$

$$\Rightarrow x^2 + nx + \frac{2(n+1)(n-1)}{6} - 10 = 0$$

$$\Rightarrow x^2 + nx + \frac{n^2-1}{3} - 10 = 0$$

let roots be $\alpha, \alpha+1$

$\Rightarrow \alpha + \alpha + 1 = -n \Rightarrow 2\alpha = -n-1 \Rightarrow \alpha = \frac{-(n+1)}{2}$

$\alpha(\alpha+1) = \frac{n^2-31}{3}$

Sub. α $\Rightarrow \left[-\frac{(n+1)}{2}\right]\left[-\frac{(n+1)}{2}+1\right] = \frac{n^2-31}{3}$

$\Rightarrow \left[-\left(\frac{n+1}{2}\right)\right]\left[-\left(\frac{n-1}{2}\right)\right] = \frac{n^2-31}{3}$

$\Rightarrow \frac{n^2-1}{4} = \frac{n^2-31}{3}$

$\Rightarrow 3n^2-3 = 4n^2-124 \Rightarrow n^2 = 121$

$\Rightarrow n = 11$

$\Rightarrow$ (3) is correct.

(63) $2\omega+1 = z \Rightarrow 2\omega = +\sqrt{-3}-1$

$\Rightarrow \omega = -\frac{1}{2}+i\sqrt{3}/2$

$\Rightarrow \omega$ is complex cube root of unity

so $1+\omega+\omega^2 = 0$ & $\omega^3 = 1$

Hence $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 1(\omega^2-\omega^4)-1(\omega-\omega^2)+1(\omega^2-\omega)$

$= \omega^2-\omega-\omega+\omega^2+\omega^2-\omega = 3(\omega^2-\omega)$

$= 3\left[-\frac{1}{2}-i\sqrt{3}/2-\left(-\frac{1}{2}+i\sqrt{3}/2\right)\right] = 3[-i\sqrt{3}]$

$= -3z$

so $3k = 3z \Rightarrow k = -z$

$\Rightarrow$ is correct.

(64) $A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$3A^2+12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{adj}(3A^2+12A) = \begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}^T = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

$\Rightarrow$ (1) is correct.

(65) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 1(a-b)-1(1-a)+1(b-a^2)$

$= 2a-1-a^2 = -[a^2-2a+1]$

if $\Delta = 0 \Rightarrow -(a-1)^2 = 0 \Rightarrow a = 1$

when $a = 1$ first & second ~~equati~~ equations are same

& third is $x + by + z = 0$

For no solution third & first should be contradictory to each other.

$\Rightarrow$ $x + y + z = 1$ & $x + by + z = 0$ are contradictory

$\Rightarrow \frac{1}{1} = \frac{1}{b} = \frac{z}{1} \neq \frac{1}{0} \Rightarrow b = 1$

so set S is singleton

$\Rightarrow$ (3) is correct.

(66)

Possible cases.

Friends of X		Friends of y		no of ways	
Ladies(4)	men(3)	Ladies(3)	men(4)		
3	0	0	3	${}^4C_3 \times {}^4C_3$	16
2	1	1	2	${}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2$	324
1	2	2	1	${}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1$	144
0	3	3	0	${}^3C_3 \times {}^3C_3$	1

Total = 485

$\Rightarrow$ (4) is correct.

(67) Given quantity can be grouped as

$${}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \text{---} {}^{21}C_{10} - ({}^{10}C_1 + {}^{10}C_2 + \text{---} {}^{10}C_{10})$$

$${}^{21}C_1 + {}^{21}C_2 + \text{---} + {}^{21}C_{10} - (2^{10} - {}^{10}C_0)$$

we ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \text{---} {}^{21}C_{10} = \frac{2^{21}}{2}$

$$\Rightarrow \left(\frac{2^{21}}{2} - 1\right) - (2^{10} - 1) = 2^{20} - 2^{10}$$

$\Rightarrow$ (3) is correct.

(68) $\Rightarrow (3\times5)^2a^2 + (3b)^2 + (5c)^2 = (3\times5)a(3b) + (3b)(5c) + (5a)(5c)$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 = (15a)(3b) + (3b)(5c) + (15a)(5c)$

$\Rightarrow 15a = 3b = 5c$

let $a = k/15$, $b = k/3$ & $c = k/5$

$\Rightarrow 2c = a+b \Rightarrow a, c, b$ are in A.P.

or b, c, a are in A.P.

$\Rightarrow$ (1) is correct.

(69) $f(x) = ax^2 + bx + c$ & $a+b+c = 1$

$f(1) = a+b+c = 3$

sub $y = 1$

$\Rightarrow f(x+1) = f(x) + f(1) + x$

$\Rightarrow f(x+1) - f(x) = 3 + x$

Now take summation on both sides from 1 to n-1

$$\sum_{x=1}^{n-1} [f(x+1) - f(x)] = \sum_{x=1}^{n-1} (3 + x)$$

$$\Rightarrow f(n) - f(1) = 3(n-1) + \frac{(n-1)n}{2}$$

$$\Rightarrow f(n) = 3 + 3n - 3 + \frac{n^2 - n}{2} = \frac{n^2}{2} + \frac{5n}{2}$$

$$\sum_{n=1}^{10} f(n) = \sum_{n=1}^{10} \frac{n^2}{2} + \sum_{n=1}^{10} \frac{5n}{2}$$

$$= \frac{1}{2} \times \frac{1}{6} \times 10 \times 11 \times 21 + \frac{5}{2} \times \frac{10 \times 11}{2} = 330$$

(70) $$\lim_{x \to \pi/2} \frac{\frac{\cos x}{\sin x} - \cos x}{(\pi - 2x)^3} = \lim_{x \to \pi/2} \frac{\cos x [1 - \sin x]}{\sin x \; (\pi - 2x)^3}$$

$$= \lim_{x \to \pi/2} \frac{\sin(\pi/2 - x)\left[1 - \cos(\pi/2 - x)\right]}{\sin x \; (\pi - 2x)^3}$$

$$= \lim_{x \to \pi/2} \frac{\sin(\pi/2 - x)}{2(\pi/2 - x)\sin x} \; \frac{2\sin^2(\pi/4 - x/2)}{(\pi - 2x)^2}$$

$$= \lim_{x \to \pi/2} \frac{2\sin(\pi/2 - x)}{2\sin x \, (\pi/2 - x)} \times \frac{\sin^2(\pi/4 - x/2)}{4^2 (\pi/4 - x/2)^2} = \frac{1}{16}$$

(71) $\tan^{-1}\dfrac{2\cdot 3x^{3/2}}{1-(3x^{3/2})^2} = 2\tan^{-1}(3x^{3/2})$

$\dfrac{d}{dx}\, 2\tan^{-1}(3x^{3/2}) = \dfrac{2}{1+(3x^{3/2})^2}\cdot 3\cdot \tfrac{3}{2}\, x^{1/2}$

$= \dfrac{9\sqrt{x}}{1+9x^3} = \dfrac{9}{1+9x^3}\cdot\sqrt{x}$

$\Rightarrow$ (4) is correct.

(72) Intersection with y axis $\Rightarrow x = 0$

$\Rightarrow y = 1$

So normal to $(0,1)$

$y(x-2)(x-3) = x+6$

diff. w.r. to $x \Rightarrow \dfrac{dy}{dx}(x-2)(x-3) + y(x-3) + y(x-2) = 1$

sub $x=0$ & $y=1$

$\Rightarrow [y'(0)](+6) - 3 - 2 = 1 \Rightarrow y'(0) = 1$

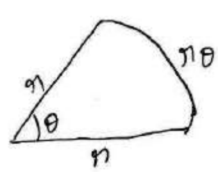
so slope of Normal $= -\dfrac{1}{1} = -1$

Eqn. of normal $y-1 = -1(x-0)$

$\Rightarrow x+y = 1$ so passes through $(\tfrac{1}{2}, \tfrac{1}{2})$

$\Rightarrow$ (a) is correct.

(73)



let radius of sector be r & angle θ

$\Rightarrow 2r + r\theta = 20 \Rightarrow \theta = \dfrac{20}{r} - 2$

Area $= \tfrac{1}{2} r^2 \theta$

$= \tfrac{1}{2} r^2 \left(\dfrac{20}{r} - 2\right)$

$= \tfrac{1}{2}\left[20r - 2r^2\right]$

$$\frac{dA}{dr} = \frac{1}{2}[20-4r] = 0 \Rightarrow r = 5$$

$$\frac{d^2A}{dr^2} < 0$$

so $A_{max} = A(5) = \frac{1}{2}\left[20\times 5 - 2\times 5^2\right] = 25\ m^2$

$\Rightarrow$ (2) is correct.

(74) $I_4 + I_6 = \int(\tan^4 x + \tan^6 x)\,dx = \int \tan^4 x \sec^2 x\,dx$

$$= \frac{\tan^5 x}{5} + c$$

so $a = \frac{1}{5}$, $b = 0$

$\Rightarrow$ (1) is correct.

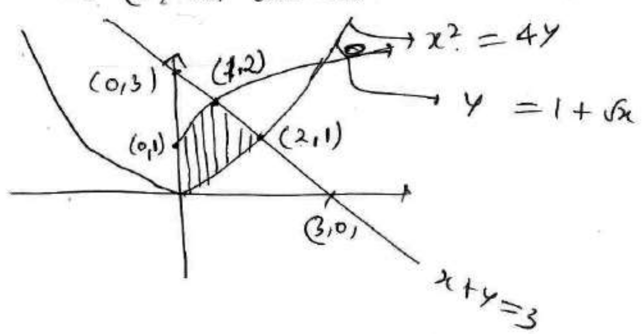
(75) $\int_{\pi/4}^{3\pi/4} \frac{dx}{2\cos^2 x/2} = \frac{1}{2}\int_{\pi/4}^{3\pi/4} \sec^2 x/2\,dx$

$$= \frac{1}{2}\left(2\tan x/2\right)\Big|_{\pi/4}^{3\pi/4}$$

$$= \tan 3\pi/8 - \tan \pi/4 = \sqrt{2}+1-(\sqrt{2}-1) = 2$$

$\Rightarrow$ (1) is correct.

(76)



Shaded portion is required region

$$\text{Area} = \int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)\,dx - \int_0^2 \frac{x^2}{4}\,dx$$

$$= 1 + \frac{2}{3} + 3 - \left(\frac{4-1}{2}\right) - \frac{8}{4\times 3}$$

$$= 1 + \frac{2}{3} + 3 - \frac{3}{2} - \frac{2}{3} = 4 - \frac{3}{2} = \frac{5}{2}$$

$\Rightarrow$ (3) is correct.

(77) $\Rightarrow \int \frac{dy}{y+1} = -\int \frac{\cos x\,dx}{\sin x+2}$

$\Rightarrow \ln(y+1) = -\ln(\sin x+2) + \ln C$

$\Rightarrow y+1 = \frac{C}{\sin x+2}$

$y(0) = 1 \Rightarrow 2 = \frac{C}{0+2} \Rightarrow C = 4$

$y = \frac{4}{\sin x+2} - 1 \Rightarrow y(\pi/2) = \frac{4}{3} - 1 = \frac{1}{3}$

(78) Area $= \left|\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ k & 5 & -k \\ -3k & k & 2 \end{vmatrix}\right| = 28$

$\Rightarrow 1(10+k^2) - 1(2k-3k^2) + 1(k^2+15k) = \pm 56$

$\Rightarrow 5k^2+13k+10 = \pm 56$

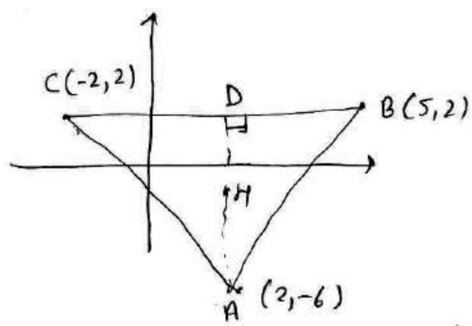
$\Rightarrow 5k^2+13k-46 = 0$ or $5k^2+13k+66 = 0$ → no real sol.

$\Rightarrow (5k+23)(k-2) = 0$

$\Rightarrow k = 2, -23/5$

k is integer $\Rightarrow k = 2$

so points are $(2,-6)$, $(5,2)$, $(-2,2)$



BC is || to x axis
so altitude from A to BC is $x = 2$

let orthocenter be $H(2,\beta)$

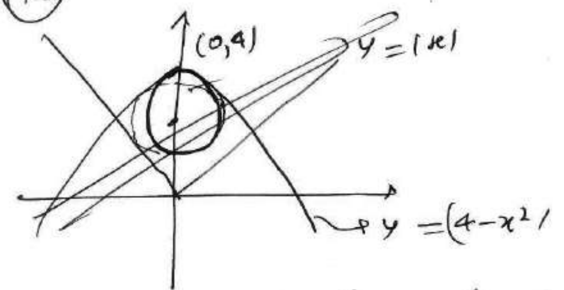
$m_{CH} \times m_{AB} = -1$

$\Rightarrow \left(\frac{\beta-2}{2+2}\right) \times \left(\frac{2+6}{5-2}\right) = -1 \Rightarrow \left(\frac{\beta-2}{4}\right)\cdot\frac{8}{3} = -1$

$\Rightarrow \beta - 2 = -3/2 \Rightarrow \beta = 1/2$

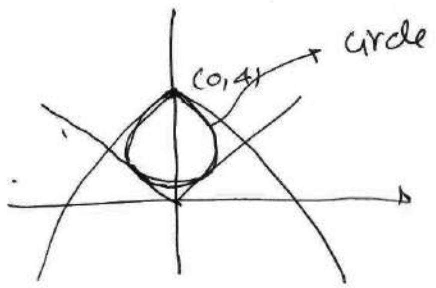
Hence (3) is correct.

(79)



Since both the curves are symmetric about y axis so circle is also symmetric about y axis

let center of circle be $(0,k)$
circle has smallest radius so distace of center of circle from parabola should be minimum ⇒ center lies on the normal to parabola ⇒ least distance of center to parabola is radius of circle = distance between vertex of parabola & center.



$\Rightarrow$ radius $= 4-k$
$\quad\quad\quad \parallel$
$\perp$ distace from center to tangent $y=x$

$$= \frac{|k-0|}{\sqrt{1+1}} = \frac{k}{\sqrt{2}}$$

$$\Rightarrow \frac{k}{\sqrt{2}} = 4-k \Rightarrow k = \frac{4\sqrt{2}}{\sqrt{2}+1}$$

$$= 4\sqrt{2}(\sqrt{2}-1)$$

$$\text{radius} = \frac{k}{\sqrt{2}} = \frac{4\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}} = 4(\sqrt{2}-1)$$

⇒ (2) is correct.

(80) $e = \frac{1}{2}$, Directrix $x = \pm \frac{a}{e} \Rightarrow x = \pm 2a$

so $a = 2$

$b^2 = a^2(1-e^2) = 4(1-\frac{1}{4}) = 3$

Eqn of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Eqn of tangent at $(1, 3/2)$ is

$$\frac{x}{4} + \frac{3}{2}\cdot\frac{y}{3} = 1 \Rightarrow \frac{x}{4} + \frac{y}{2} = 1$$

$\Rightarrow x + 2y = 4$

so eqn of normal is $2x - y + c = 0$

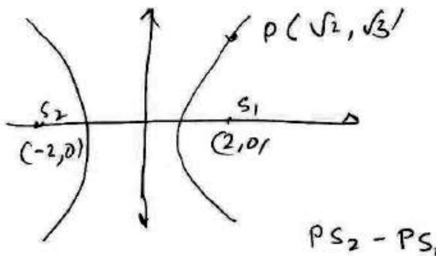
passes through $1, 3/2$

$\Rightarrow 2 - 3/2 + c = 0 \Rightarrow c = -\frac{1}{2}$

so eqn is $2x - y - \frac{1}{2} = 0 \Rightarrow 4x - 2y - 1 = 0$

$\Rightarrow$ (1) is correct.

(81)



$PS_2 = \sqrt{(\sqrt{2}+2)^2 + 3}$

$= \sqrt{9 + 4\sqrt{2}} = \sqrt{(\sqrt{8}+1)^2}$

$PS_1 = \sqrt{9 - 4\sqrt{2}} = \sqrt{(\sqrt{8}-1)^2}$

$PS_2 - PS_1 = 2\sqrt{2} + 1 - (2\sqrt{2} - 1)$

$= 2 = 2a$

$\Rightarrow a = 1$

focus $(\pm ae, 0) = (\pm 2, 0) \Rightarrow ae = 2 \Rightarrow e = 2$

$b^2 = 1(2^2 - 1) = 3$

Eqn. of hyperbola. $x^2 - \frac{y^2}{3} = 1$

Eqn of tangent at $(\sqrt{2}, \sqrt{3})$ is

$\sqrt{2}x - \frac{\sqrt{3}y}{3} = 1 \Rightarrow \sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

passes through $(2\sqrt{2}, 3\sqrt{3})$

$\Rightarrow$ (1) is correct.

(82) $\bar{n} \perp \hat{i} - 2\hat{j} + 3\hat{k}$ & $2\hat{i} - \hat{j} - \hat{k}$

$$\Rightarrow n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{j} + 3\hat{k}$$

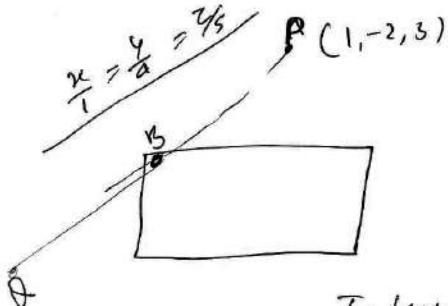
Eqn of plane $\Rightarrow r \cdot (5\hat{i} + 7\hat{j} + 3\hat{k}) = (\hat{i} - \hat{j} - \hat{k}) \cdot (5\hat{i} + 7\hat{j} + 3\hat{k})$

$\Rightarrow 5x + 7y + 3z = 5 - 7 - 3$

$\Rightarrow 5x + 7y + 3z + 5 = 0$

$\perp$ distance from $(1, 3, -7) = \frac{|5 + 21 - 21 + 5|}{\sqrt{25 + 49 + 9}} = \frac{10}{\sqrt{83}}$

(83) distance of image of $(1,-2,3)$ from plane is same as that of point from the plane mesaured $\parallel$ to given line



Eqn of line AB

$$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$$ ($\parallel$ to given line & passes through $(1,-2,3)$)

Intersection with plane

let B be $(\lambda+1, 4\lambda-2, 5\lambda+3)$.

B lies on the plane

$\Rightarrow 2(\lambda+1) + 3(4\lambda-2) - 4(5\lambda+3) + 22 = 0$

$-6\lambda + 6 = 0 \Rightarrow \lambda = 1$

so B $= (2,2,8)$

PB $= \sqrt{1+16+25} = \sqrt{42} \Rightarrow PQ = 2PB = 2\sqrt{42}$

$\Rightarrow$ (2) is correct.

(84) $|\bar{c}-\bar{a}| = 3 \Rightarrow |c|^2 + |a|^2 - 2\bar{c}\cdot\bar{a} = 9$

$\Rightarrow |c|^2 + 9 - 2\bar{c}\cdot\bar{a} = 9$ — (1)

$|(\bar{a}\times\bar{b})\times c| = 3$

$\Rightarrow |\bar{a}\times\bar{b}|\,|c| \sin 30^\circ = 3$

$\Rightarrow \sqrt{4+4+1}\,|c| \times \frac{1}{2} = 3$

$\Rightarrow |c| = 2$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} + 2\hat{j} + \hat{k}$$

sub $|c|$ in (1) $\Rightarrow 4 + 9 - 2\cdot\bar{c}\cdot\bar{a} = 9$

$\Rightarrow \bar{c}\cdot\bar{a} = 2$

$\Rightarrow$ (1) is correct.

(85) ~~It is~~ Experiment is ~~best~~ bernouli's trial with $p = \frac{15}{25}$ (drawing of green ball)

$= 3/5$

$q = 2/5$, $n = 10$

Variance of Bernouli's trial $= npq$

$= 10 \times \frac{3}{5} \times \frac{2}{5}$

$= \frac{12}{5}$

$\Rightarrow (4)$

(86) $P(A \cup B) - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ — (1)

similarly $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ — (2)

$P(C) + P(A) - 2P(A \cap C) = \frac{1}{4}$ — (3)

Adding (1), (2), (3)

$\Rightarrow 2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{4}$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(C \cap A)] + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

(87) Total no. of ways $= {}^{11}C_2 = \frac{11 \times 10}{2} = 55$

If sum & diff. of no. is ~~divisible~~ multiple of 4

$\Rightarrow$ both are multiple of 4 or both are $4n+2$ form

(0, 4, 8) (2, 6, 10)

$\Rightarrow {}^3C_2 + {}^3C_2 = 6$

So pro. $= \frac{6}{55} \Rightarrow (4)$

(88) $5\left[\sec^2 x - 1 - \cos^2 x\right] = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\sec^2 x - 9\cos^2 x = 12$

$\Rightarrow \frac{5}{\cos^2 x} - 9\cos^2 x = 12 \Rightarrow 5 - 9\cos^4 x = 12\cos^2 x$

$\Rightarrow 9\cos^4 x + 12\cos^2 x - 5 = 0$

$\Rightarrow (3\cos^2 x + 5)(3\cos^2 x - 1) = 0$

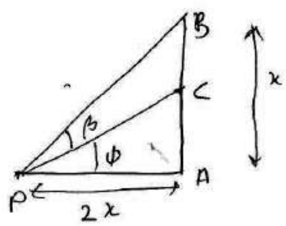
$\Rightarrow \cos^2 x = \frac{1}{3}$

$\Rightarrow \cos 2x = 2\times\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

$\Rightarrow$ (4) is correct.

(89)



Let $\angle CPA = \phi$

$\Rightarrow \tan\phi = \frac{x/2}{2x} = \frac{1}{4}$

$\tan(\phi + \beta) = \frac{x}{2x} = \frac{1}{2}$

$\Rightarrow \frac{\tan\phi + \tan\beta}{1 - \tan\phi\tan\beta} = \frac{1}{2}$

$\Rightarrow \frac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} = \frac{1}{2} \Rightarrow \frac{1}{2} + 2\tan\beta = 1 - \frac{1}{4}\tan\beta$

$\Rightarrow \frac{9}{4}\tan\beta = \frac{1}{2}$

$\Rightarrow \tan\beta = \frac{2}{9}$

$\Rightarrow$ (2) is correct.

(90)

p	q	$p \to q$	$\sim p \to q$	$(\sim p \to q) \to q$	$(p \to q) \to [((\sim p) \to q) \to q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

so given statement is tautology.

so (4) is correct.