Indefinite Integration

1

Q1 $\int \tan^4 x\,dx$

$$= \int \tan^2 x \tan^2 x\,dx$$
$$= \int \tan^2 x(\sec^2 x - 1)\,dx$$
$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$$
$$= \frac{\tan^3 x}{3} - \int(\sec^2 x - 1)\,dx$$
$$= \frac{\tan^3 x}{3} - \tan x + x + c$$

Q2 $\int(\sin 2x - \cos 2x)\,dx = -\frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x + c$

$$= -\frac{1}{2}[\cos 2x + \sin 2x] + c = \cancel{-\frac{1}{2}\cdot\sqrt{2}\cos\left(2x-\frac{\pi}{4}\right)}$$

$$= -\frac{1}{2}\cdot\sqrt{2}\sin(2x + \pi/4) + c$$

Now $-\frac{1}{\sqrt{2}}\sin(2x+\pi/4) = \frac{1}{\sqrt{2}}\sin(2x-a)$

$\Rightarrow \sin(2x+\pi/4) = -\sin(2x-a)$

$\Rightarrow 2x + \pi/4 = (2n+1)\pi + 2x - a, \quad n \in I$

$\Rightarrow a = (2n+1)\pi - \pi/4, \quad n \in I$

Q3 $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx = \int e^x\left(\frac{1 + 2\sin x/2\cos x/2}{2\cos^2 x/2}\right)dx$

$$= \int e^x\left(\frac{1}{2}\sec^2 x/2 + \tan x/2\right)dx$$
$$= e^x \tan x/2 + c$$

Q4 $\int \frac{\cos 4x + 1}{\cot x - \tan x}\,dx = \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}\,dx$

$$= \int \frac{2\cos^2 2x \sin x \cos x}{\cos^2 x - \sin^2 x}\,dx$$
$$= \int \frac{(\cos^2 2x)\sin 2x}{\cos 2x}\,dx = \int \cos 2x \sin 2x\,dx$$
$$= \frac{1}{2}\int \sin 4x\,dx$$
$$= -\frac{1}{8}\cos 4x + c$$

(5) $\int x^x(1+\ln x)\,dx$ let $x^x = t$ (2)

$= \int dt$ $\Rightarrow x^x(1+\ln x)\,dx = dt$

$= t + c = x^x + c$

(6) $\int x^{-2/3}\left(1+x^{1/2}\right)^{-5/3}dx = \int x^{-2/3}\left(x^{1/2}\right)^{-5/3}\left(x^{-1/2}+1\right)^{-5/3}dx$

$= \int x^{-2/3}\,x^{-5/6}\left(x^{-1/2}+1\right)^{-5/3}dx$

$= \int x^{-3/2}\left(x^{-1/2}+1\right)^{-5/3}dx$

Now sub. $x^{-1/2}+1 = t$

(7) $f''(x) = \sec^4 x + 4$

$f'(x) = \int \sec^4 x\,dx + \int 4\,dx$

$= \int \sec^2 x\,(1+\tan^2 x)\,dx + 4x + c$

$= \tan x + \dfrac{\tan^3 x}{3} + 4x + c$

$f'(0) = 0 \Rightarrow c = 0$

$f'(x) = \tan x + \dfrac{\tan^3 x}{3} + 4x + 0$

$f(x) = \int \tan x\,dx + \frac{1}{3}\int \tan^3 x\,dx + \int 4x\,dx$

$= \ln|\sec x| + \frac{1}{3}\int (\sec^2 x - 1)\tan x\,dx + 2x^2 + c_1$

$= \ln|\sec x| + \frac{1}{3}\int \tan x \sec^2 x\,dx - \frac{1}{3}\int \tan x\,dx + 2x^2 + c_1$

$= \ln|\sec x| + \frac{1}{6}\tan^2 x - \frac{1}{3}\ln|\sec x| + 2x^2 + c_1$

$f(x) = \frac{2}{3}\ln|\sec x| + \frac{1}{6}\tan^2 x + 2x^2 + c_1$

$f(0) = 0 \Rightarrow c_1 = 0$

(8) $\int \dfrac{3x+4}{x^3-2x-4}\,dx = \int \dfrac{3x+4}{(x-2)(x^2+2x+2)}\,dx$

$\dfrac{3x+4}{(x-2)(x^2+2x+2)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2x+2}$

$$\Rightarrow 3x+4 = A(x^2+2x+2) + (Bx+C)(x-2) \quad (3)$$

sub. $x = 2 \Rightarrow 10 = A(10) \Rightarrow A = 1$

sub. $x = 0$

$4 = 2A - 2C \Rightarrow C = -1$

sub. $x = 1$

$7 = 5A + (B+C)(-1) \Rightarrow B = -1$

so $\displaystyle\int \frac{3x+4}{(x-2)(x^2+2x+2)}dx = \int \frac{1}{x-2}dx + \int \frac{-1-x}{x^2+2x+2}dx$

$= \ln|x-2| - \frac{1}{2}\ln|x^2+2x+2|$

(9) $\displaystyle\int [x\, l(x)\, l^2(x)\, l^3(x) \cdots l^r(x)]^{-1} dx$

let $l^{r+1}(x) = t = \ln\ln\ln\ln \cdots$ upto r+1 times $\ln x$

$\Rightarrow dt = \frac{1}{l^r(x)} \cdot \frac{1}{l^{r-1}(x)} \cdot \frac{1}{l^{r-2}(x)} \cdots \frac{1}{l(x)} \cdot \frac{1}{x} dx$

$\Rightarrow dt = [l^r(x)\, l^{r-1}(x)\, l^{r-2}(x) \cdots l(x) \cdot x]^{-1} dx$

So given integral is

$\int dt = t + c = l^{r+1}(x) + c$

(10) $\displaystyle\int \frac{xe^x}{(1+x)^2}dx = \int e^x\left(\frac{x+1-1}{(1+x)^2}\right)dx$

$= \displaystyle\int e^x\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]dx = \frac{e^x}{1+x} + c$

(11) $\displaystyle\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx$ let $x^{3/2} = t$

$\frac{3}{2}x^{1/2}dx = dt$

$= \displaystyle\int \frac{\frac{2}{3}dt}{\sqrt{1-t^2}} = \frac{2}{3}\sin^{-1}t + c$

(12) $\displaystyle\int 5^x \cdot 5^{5^x}\, 5^{5^{5^x}} dx$ let $5^{5^{5^x}} = t$

$\Rightarrow 5^{5^{5^x}} \ln 5 \cdot 5^{5^x} \ln 5 \cdot 5^x \ln 5\, dx = dt$

$\Rightarrow 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x (\ln 5)^3 dx = dt$

$\Rightarrow \displaystyle\int \frac{dt}{(\ln 5)^3} = \frac{t}{(\ln 5)^3} + c$

(13) $\int \operatorname{cosec} 2x\, dx = \frac{1}{2} \ln|\tan x| + c = f\{g(x)\} + c$ (4)

$\Rightarrow \quad g(x) = \tan x \quad \& \quad f(x) = \frac{1}{2}\ln|x|$

(14) $\int \frac{x^5}{\sqrt{1+x^3}}\, dx = \int \frac{x^3 \cdot x^2}{\sqrt{1+x^3}}\, dx$

let $1 + x^3 = t^2$

$3x^2\, dx = 2t\, dt$

$\int \frac{(t^2-1)}{\sqrt{t^2}} \frac{2t\, dt}{3} = \frac{2}{3}\int (t^2 - 1)\, dt$

(15) $\int \tan^3 2x \sec 2x\, dx$ let $\sec 2x = t$

$2\sec 2x \tan 2x\, dx = dt$

$\int \tan^2 2x \cdot \frac{dt}{2} = \frac{1}{2}\int (\sec^2 2x - 1)\, dt$

$= \frac{1}{2}\int (t^2 - 1)\, dt$

(16) $\int \frac{x^5}{\sqrt{1+x^2}}\, dx = \int \frac{(x^2)^2\, x}{\sqrt{1+x^2}}\, dx$

Sub. $1 + x^2 = t^2$

(17) $\int x \sin^2 x\, dx = \int x \frac{(1-\cos 2x)}{2}\, dx = \frac{x^2}{4} - \frac{1}{2}\int \underset{\uparrow \text{ integration by parts}}{x \cos 2x\, dx}$

$\int \underset{I^{st}}{x}\ \underset{2^{nd}}{\cos 2x}\, dx = \cancel{\cos}\ x \int \cos 2x\, dx - \int \left[\frac{d}{dx} x \cdot \int \cos 2x\, dx\right] dx$

$= \frac{1}{2} x \sin 2x - \int \frac{1}{2}\sin 2x\, dx$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x$

(18) $\int \cos\sqrt{x}\, dx$ let $x = t^2$

$dx = 2t\, dt$

$\int \cos t\, (2t\, dt) = 2\int t \cos t\, dt$ (Now integrate by parts)

(19) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx = \int \frac{\sin^{-1}\sqrt{x} - \left(\pi/2 - \sin^{-1}\sqrt{x}\right)}{\pi/2}\, dx$

$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\, dx - \int dx$

(5)

Now $\int \sin^{-1}\sqrt{x}\, dx$ let $x = \sin^2\theta$, $dx = d(\sin^2\theta)$

$$\Rightarrow \int \theta \, d(\sin^2\theta) = \theta \sin^2\theta - \int 1 \times \sin^2\theta \, d\theta$$

$$= \theta \sin^2\theta - \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \theta \sin^2\theta - \theta/2 + \frac{1}{4}\sin 2\theta + c$$

$$= x \sin^{-1}\sqrt{x} - \frac{1}{2}\sin^{-1}\sqrt{x} + \frac{1}{2}\sin\theta\cos\theta + c$$

$$= x \sin^{-1}\sqrt{x} - \frac{1}{2}\sin^{-1}\sqrt{x} + \frac{1}{2}\sqrt{x}\sqrt{1-x} + c$$

(20) $\int (1-\cos x)\operatorname{cosec}^2 x \, dx = \int \frac{2\sin^2 x/2}{\sin^2 x} dx = \int \frac{2\sin^2 x/2}{(2\sin x/2 \cos x/2)^2} dx$

$$= \frac{1}{2}\int \sec^2 x/2 \, dx$$

(21) $\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos(\pi/2 - x)} = \int \frac{dx}{2\cos^2(\pi/4 - x/2)}$

$$= \frac{1}{2}\int \sec^2(\pi/4 - x/2)\, dx$$

$$= \frac{\frac{1}{2}}{-\frac{1}{2}} \tan\left(\pi/4 - x/2\right) + c$$

(22) Sub $1+\ln x = t$

(23) $\int \frac{4e^x + \frac{6}{e^x}}{9e^x - 4/e^x} dx = \int \frac{4e^{2x}+6}{9e^{2x}-4} dx$

let $4e^{2x}+6 = A(9e^{2x}-4) + B\frac{d}{dx}(9e^{2x}-4)$

$$= A(9e^{2x}-4) + B(18e^{2x})$$

Now comparing coeff of e^{2x} & constant term

$\Rightarrow 4 = 9A + 18B$ & $-4A = 6 \Rightarrow A = -3/2,\ B = \frac{35}{36}$

so $\int \frac{4e^{2x}+6}{9e^{2x}-4} dx = -3/2 \int \frac{9e^{2x}-4}{9e^{2x}-4} dx + \frac{35}{36}\int \frac{18e^{2x}}{9e^{2x}-4} dx$

$$= -3/2\, x + \frac{35}{36}\ln|9e^{2x}-4| + c$$

(24) $\int \frac{1}{x(x^3+1)} dx = \int \frac{1}{x^4(1+1/x^3)} dx$ now sub. $1 + 1/x^3 = t$

(25) $\int \frac{1+x^4}{(1-x^4)^{3/2}}\,dx = \int \frac{1+x^4}{(x^2)^{3/2}\left(\frac{1}{x^2}-x^2\right)^{3/2}}\,dx = \int \frac{\frac{1}{x^3}+x}{\left(\frac{1}{x^2}-x^2\right)^{3/2}}\,dx$ (6)

Now sub. $\frac{1}{x^2}-x^2 = t$

$\Rightarrow -2\left(\frac{1}{x^3}+x\right)dx = dt$

$\Rightarrow$ given integration is $\int \frac{-\frac{1}{2}\,dt}{t^{3/2}} = -\frac{1}{2}\frac{t^{-1/2}}{-\frac{1}{2}} + c$

$= \left(\frac{1}{x^2}-x^2\right)^{-1/2} + c$

(26) $\int \left(x^{\sin x-1}\sin x + x^{\sin x}\cos x\ln x\right)dx = \int x^{\sin x}\left(\frac{\sin x}{x} + \cos x\ln x\right)dx$

Now sub. $x^{\sin x} = t$

$\Rightarrow x^{\sin x}\left[\frac{\sin x}{x} + \cos x\ln x\right]dx = dt$

$\Rightarrow \int dt = t + c = x^{\sin x} + c$

(27) $\int \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{\sqrt{1+x^2}}\,dx$ let $x+\sqrt{1+x^2} = t$

$\left(1+\frac{1\times 2x}{2\sqrt{1+x^2}}\right)dx = dt$

$\Rightarrow \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)dx = dt$

$\Rightarrow \frac{t\,dx}{\sqrt{1+x^2}} = dt$

$\Rightarrow \int \frac{t^{15}\,dt}{t}$

$= \int t^{14}dt = \frac{t^{15}}{15} + c$

$= \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{15} + c$

(28)

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(28) $\int \frac{dx}{\sin x \sin(x+\alpha)} = \int \frac{dx}{\sin x[\sin x \cos\alpha + \cos x \sin\alpha]}$ (7)

$= \int \frac{dx}{\sin^2 x\left[\cos\alpha + \frac{\cos x}{\sin x}\sin\alpha\right]} = \int \frac{\operatorname{cosec}^2 x \, dx}{[\cos\alpha + \cot x \sin\alpha]}$

Now sub. $\cos\alpha + \cot x \sin\alpha = t$

$\Rightarrow -\sin\alpha \operatorname{cosec}^2 x \, dx = dt$

So given integration reduces to $\int \frac{-dt}{\sin\alpha\,(t)}$

(29) Sub. $\sqrt{x} = t$

(30) $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx = \int \frac{(x^2-1)\,dx}{x^3 x^2\sqrt{2 - \frac{2x^2}{x^4} + \frac{1}{x^4}}}$

$= \int \frac{\frac{x^2}{x^5} - \frac{1}{x^5}}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}dx$ let $2 - \frac{2}{x^2} + \frac{1}{x^4} = t$

$\Rightarrow \left(-\frac{4}{x^3} - \frac{4}{x^5}\right)dx = dt$

$\Rightarrow \int \frac{-\frac{1}{4}dt}{\sqrt{t}}$

(31) $I + J = \int \left[\frac{e^x}{e^{4x} + e^{2x}+1} + \frac{e^{-x}}{e^{-4x} + e^{-2x}+1}\right]dx$

$= \int \left[\frac{e^x}{e^{4x} + e^{2x}+1} + \frac{\frac{1}{e^x}}{\frac{1}{e^{4x}} + \frac{1}{e^{2x}}+1}\right]dx$

$= \int \frac{e^x + e^{3x}}{e^{4x} + e^{2x}+1}dx$ let $e^x = t$

$e^x dx = dt$

$\Rightarrow \int \frac{e^x(e^{2x}+1)}{e^{4x} + e^{2x}+1}dx$

$= \int \frac{t^2+1}{t^4+t^2+1}dt = \int \frac{1+\frac{1}{t^2}}{t^2+1+\frac{1}{t^2}}dt$

$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+3}$

Now Sub $t - \frac{1}{t} = u$

(32) $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}}\,dx = \int \frac{\cos^{3/2} x}{\sin^{11/2} x}\,dx = \int \frac{\cos^{3/2} x}{\sin^{7/2} x \sin^2 x}\,dx$ (8)

$= \int \frac{\cos^{3/2} x \quad \operatorname{cosec}^2 x}{\sin^{3/2} x \sin^2 x}\,dx = \int \cot^{3/2} x \operatorname{cosec}^2 x \operatorname{cosec}^2 x\,dx$

$= \int \cot^{3/2} x\,(1+\cot^2 x) \operatorname{cosec}^2 x\,dx$

$= \int \left(\cot^{3/2} x + \cot^{7/2} x\right) \operatorname{cosec}^2 x\,dx$

now sub $\cot x = t \Rightarrow -\operatorname{cosec}^2 x\,dx = dt$

$= \int \left(t^{3/2} + t^{7/2}\right)(-dt)$

$= \frac{-2t^{5/2}}{5} - \frac{2t^{9/2}}{9} + c = -\frac{2}{5}\cot^{5/2} x - \frac{2}{9}\cot^{9/2} x + c$

$= -\frac{2}{5}\tan^{-5/2} x - \frac{9}{5}\tan^{-9/2} x + c$

(33) $\int \frac{\sec x\,dx}{(\sec x + \tan x)^2} = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)^3}\,dx$

$\sec x + \tan x = t$

$(\sec x \tan x + \sec^2 x)\,dx = dt \Rightarrow \sec x(\tan x + \sec x)\,dx = dt$

$\Rightarrow \int \frac{dt}{t^3}$

(34) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \times \sqrt{\frac{1-\sqrt{x}}{1-\sqrt{x}}}\,dx = \int \frac{1-\sqrt{x}}{\sqrt{1-x}}\,dx$

$\int \left(\frac{1}{\sqrt{1-x}} - \frac{\sqrt{x}}{\sqrt{1-x}}\right)dx = -2\sqrt{1-x} - \int \frac{\sqrt{x}}{\sqrt{1-x}} \frac{\sqrt{x}}{\times \sqrt{x}}\,dx$

$= -2\sqrt{1-x} - \int \frac{x}{\sqrt{x-x^2}}\,dx$

integrate

(35) $\int \frac{dx}{\sin^6 x + \cos^6 x} = \int \frac{dx}{(\sin^2 x)^3 + (\cos^2 x)^3} = \int \frac{dx}{[(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x]}$

$= \int \frac{dx}{1 - 3\sin^2 x \cos^2 x} = \int \frac{dx}{1 - \frac{3}{4}\sin^2 2x} = \int \frac{4\,dx}{4 - 3\sin^2 2x}$

$= \int \frac{4\sec^2 2x\,dx}{4\sec^2 2x - 3\sin^2 2x \sec^2 2x} = \int \frac{4\sec^2 2x}{4 + \tan^2 2x}\,dx$

$$= \frac{4}{\cancel{2}} \int \frac{\sec^2 2x}{\cancel{4} + \tan^2 2x}\, dx$$

Sub $\tan 2x = t \Rightarrow 2\sec^2 2x\, dx = dt$

$$\Rightarrow \int \frac{2\,dt}{4+t^2} = \frac{2}{2}\tan^{-1}\left(\frac{t}{2}\right) + c$$

$$= \tan^{-1}\left(\frac{\tan 2x}{2}\right) + c$$

$$= \tan^{-1}\left[\frac{2\tan x}{(1-\tan^2 x)\times 2}\right] + c$$

$$= \cot^{-1}\left(\frac{1-\tan^2 x}{\tan x}\right) + c$$

$$= \cot^{-1}(\cot x - \tan x) + c$$

$$= \pi/2 - \tan^{-1}(\cot x - \tan x) + c$$

$$= -\tan^{-1}(\cot x - \tan x) + c_1$$

(37) Sub. $\ln \tan x = t$

$$\Rightarrow \frac{1}{\tan x}\sec^2 x\, dx = dt \Rightarrow \frac{1}{\sin x \cos x}\, dx = dt$$

so given integral is $\int t\, dt = \frac{(\ln \tan x)^2}{2} + c$

(38) $\int \left(\frac{\ln x - 1}{1+(\ln x)^2}\right)^2 dx$ let $\ln x = t$

$\Rightarrow x = e^t \Rightarrow dx = e^t dt$

$$\Rightarrow \int \left(\frac{t-1}{1+t^2}\right)^2 e^t dt = \int \frac{(t^2+1-2t)}{(t^2+1)^2} e^t dt = \int \left[\underbrace{\frac{1}{t^2+1}}_{f(t)} - \underbrace{\frac{2t}{(t^2+1)^2}}_{f'(t)}\right] e^t dt$$

$$= \frac{e^t}{t^2+1} + c$$

(39) $\int \frac{1}{e^x + e^{-x}}\, dx = \int \frac{e^x}{e^{2x}+1}\, dx$ Now sub. $e^x = t$

$\Rightarrow e^t dx = dt$

$$= \int \frac{dt}{t^2+1}$$

(40) $\int \frac{dx}{x^3(x^3+1)^{1/3}} = \int \frac{dx}{x^3 (x^3)^{1/3} (1+x^{-3})^{1/3}} = \int \frac{dx}{x^4 (1+x^{-3})^{1/3}}$ (10)

Now sub. $1+x^{-3} = t$

$-\frac{3}{x^4} dx = dt \Rightarrow \int \frac{-\frac{1}{3} dt}{t^{1/3}}$

(41) $\int \frac{\cos x}{\cos(x-a)} dx = \int \frac{\cos(\overline{x-a}+a)}{\cos(x-a)} dx$

$= \int \left[\frac{\cos(x-a) \cos a - \sin(x-a)\sin a}{\cos(x-a)}\right] dx$

$= \int [\cos a - \sin a \tan(x-a)] dx$

(42) $\int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

Let $\cos x + \sin x = t$

$\Rightarrow (-\sin x + \cos x) dx = dt$

$\Rightarrow$ given integral $= \int \frac{dt}{t} = \ln|\cos x + \sin x| + c.$

(43) $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}} dx = \int \sqrt{\frac{\cos x(1-\cos^2 x)}{1-\cos^3 x}} dx = \int \sqrt{\frac{\cos x \sin^2 x}{1-(\cos^{3/2} x)^2}} dx$

$\Rightarrow \int \sqrt{\frac{\cos x}{1-(\cos^{3/2} x)^2}} \cdot \sin x \, dx$ Let $\cos^{3/2} x = t$

$\frac{3}{2} \cos^{1/2} x (-\sin x) dx = dt$

$\Rightarrow \int \frac{-\frac{2}{3} dt}{\sqrt{1-t^2}} = -\frac{2}{3} \sin^{-1} t + c$

(44) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} \cos^2 x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x \, dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Now sub. $\tan x = t$

(45) $\int \frac{dx}{x^{1/5}(1+x^{4/5})^{1/2}}$ sub. $1+x^{4/5} = t$

$\frac{4}{5 x^{1/5}} dx = dt$

(11)

(46) $\int \frac{\sqrt{x}}{1+x}dx$ sub $x = t^2$, $dx = 2t\,dt$

$\Rightarrow \int \frac{t \cdot 2t\,dt}{1+t^2} = 2\int \left(\frac{t^2+1-1}{t^2+1}\right)dt = 2\int\left(1-\frac{1}{1+t^2}\right)dt$

$= 2t - 2\tan^{-1}t + c$

(47) $\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$ let $\sqrt{x}+1 = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$\int \frac{2\,dt}{t}$

(49) $I_n = \int (\ln x)^n dx$ Integrating by parts taking 1 as second function

$I_n = x(\ln x)^n - \int x\, n(\ln x)^{n-1}\cdot\frac{1}{x}dx$

$I_n = x(\ln x)^n - n\int(\ln x)^{n-1}dx$

$\Rightarrow I_n = x(\ln x)^n - nI_{n-1} \Rightarrow I_n + nI_{n-1} = x(\ln x)^n$

(50) $\int x\sqrt{\frac{1-x}{1+x}}\,dx = \int x\sqrt{\frac{1-x}{1+x}}\times\sqrt{\frac{1-x}{1-x}}\,dx = \int\frac{x(1-x)}{\sqrt{1-x^2}}dx$

$= \int\frac{x-x^2+1-1}{\sqrt{1-x^2}}dx = \int\frac{x}{\sqrt{1-x^2}}dx + \int\sqrt{1-x^2}\,dx - \int\frac{1}{\sqrt{1-x^2}}dx$

$= -\sqrt{1-x^2} + \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x + c$

(51) $\frac{1}{2}\int\frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x} = \frac{1}{2}\int\frac{dx}{\cos x\sin\pi/6 + \cos\pi/6\sin x} = \frac{1}{2}\int\frac{dx}{\sin(x+\pi/6)}$

$= \frac{1}{2}\int\operatorname{cosec}(x+\pi/6)\,dx = \frac{1}{2}\ln\left|\tan\left(\frac{x+\pi/6}{2}\right)\right| + c$

(52) $\int\frac{dx}{(x^2+1)(x^2+4)}$ let $\frac{1}{(x^2+1)(x^2+4)} = \frac{A}{x^2+1} + \frac{B}{x^2+4}$

$\Rightarrow 1 = A(x^2+4) + B(x^2+1)$

$\Rightarrow A + B = 0,\ 4A + B = 1$

$A = 1/3 = -B$

$\Rightarrow \int\frac{1}{3}\frac{dx}{x^2+1} - \frac{1}{3}\int\frac{dx}{x^2+4}$

$= \frac{1}{3}\tan^{-1}x - \frac{1}{3\times2}\tan^{-1}(x/2)$

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53 let $\frac{x^3+1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$

Find A, B, C, D & integrate

Objective assignment level 2