TOPIC BASED QUESTIONS

Level - I

Straight objective type

1. If $f(x) = \sqrt{x^2+9}$, then $\lim_{x\to 4} \frac{f(x)-f(4)}{x-4}$ has the value

(a) 5/4 (b) –4/5 (c) 4/5 (d) none of these

2. For the curve $\sqrt{x} + \sqrt{y} = 1$, dy/dx at (1/4, 1/4) is

(a) 1/2 (b) 1 (c) –1 (d) 2

3. If $f(x) = \log |x|$, $x \neq 0$ then $f'(x)$ equals

(a) 1/|x| (b) 1/x (c) –1/x (d) none of these

4. $xe^{xy} = y + \sin^2 x$ then $y'(0)$ and $y''(0)$ respectively

(a) 1, 2 (b) 1, – 2 (c) 0, – 2 (d) – 1, 2

5. Let $f(x)$ be a polynomial in x. Then the second order derivative of $f(e^x)$, is

(a) $f''(e^x) \cdot e^x + f'(e^x)$ (b) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^{2x}$

(c) $f''(e^x)\, e^{2x}$ (d) $f''(e^x)\, e^{2x} + f'(e^x) \cdot e^x$

6. Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \log_e x$. If $F(x) = (\text{hog of})(x)$, then $F''(x)$ is equal to

(a) $2\,\text{cosec}^3 x$ (b) $2 \cot x^2 - 4x^2 \text{cosec}^2 x^2$

(c) $2x \cot x^2$ (d) $-2\,\text{cosec}^2 x$

7. If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, the for $2 < x < 4$, g' equals

(a) –1 (b) 1 (c) 0 (d) none of these

8. Let $U = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $V = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{dU}{dV} =$

(a) $\frac{1}{2}$ (b) x (c) $\frac{1-x^2}{1+x^2}$ (d) 1

9. If $2^x + 2^y = 2^{x+y}$, then the value of dy/dx at $x = y = 1$ is

(a) 0 (b) –1 (c) 1 (d) 2

10. The expression of dy/dx of the function $y = a^{x^{a^{x^{\cdots\cdots\infty}}}}$ is

(a) $\frac{y^2}{x(1-y\log x)}$ (b) $\frac{y^2 \log y}{x(1-y\log x)}$ (c) $\frac{y^2 \log y}{x(1-y\log x \log y)}$ (d) $\frac{y^2 \log y}{x(1+y\log x \log y)}$

11. The differential coefficient of $f(x) = \log(\log x)$ with respect to x is

(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$ (c) $(x \log x)^{-1}$ (d) $x \log x$

12. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ then $\frac{dy}{dx}$ is

(a) 1 (b) $\frac{x-1}{x+1}$ (c) 0 (d) $\frac{x+1}{x-1}$

13. If $f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$, $0 \le x \le \frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is

(a) –1/4 (b) –1/2 (c) 1/4 (d) 1/2

14. If $y = \left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]$ (b) $\left(1+\frac{1}{x}\right)^x \log\left(1+\frac{1}{x}\right)$

(c) $\left(x+\frac{1}{x}\right)^x\left[\log(x+1)-\frac{x}{x+1}\right]$ (d) $\left(x+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{x+1}\right]$

15. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) $\sec\theta$ (d) $|\sec\theta|$

16. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

(a) $\sin(\log x).\frac{1}{x\log x}$ (b) $\frac{12}{(3-2x)^2}\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$

(c) $\sin\left(\log\left(\frac{2x+3}{3-2x}\right)\right)$ (d) none of these

17. If $y^{1/m} = \left[x+\sqrt{1+x^2}\right]$, then $(1+x^2)y_2 + xy_1$ is equal to

(a) m^2y (b) my^2 (c) m^2y^2 (d) none of these

18. If $x = \phi(t)$, $y = \psi(t)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$ (b) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$ (c) $\frac{\phi''}{\psi''}$ (d) $\frac{\psi''}{\phi''}$

19. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $f'(-3)$ equals

(a) 4 (b) 2 (c) -2 (d) 0

20. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is a constant. Then $\frac{d^3}{dx^3}\{f(x)\}$ at $x = 0$ is

(a) p (b) $p + p^2$ (c) $p + p^3$ (d) independent of p

21. If $t = e^x$ and $y = t^2 - 1$ then $(dx/dy)_{t=e}$ is

(a) $1/2e^2$ (b) ½ (c) 2 (d) $2e^2$

22. Let $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ and at $x = 1$, $y = 1$. Then $\frac{d^2y}{dx^2}$ at $y = 1$ is

(a) $-8\frac{22}{27}$ (b) $-7\frac{21}{28}$ (c) 8 (d) $\frac{22}{7}$

23. If $x^2 + y^2 = a^2$ and $k = 1/a$, then k is equal to

(a) $\frac{y''}{\sqrt{1+y'}}$ (b) $\frac{|y''|}{\sqrt{(1+y'^2)^3}}$ (c) $\frac{2y''}{\sqrt{1+y'^2}}$ (d) $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

24. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + ...\infty}}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y + x}{y^2 - 2x}$ (b) $\frac{y^2 - x}{2y^3 - 2xy - 1}$ (c) $\frac{y^3 + x}{2y^2 - x}$ (d) none of these

25. Let f and g be differentiable functions satisfying $g'(a) = 2$, $g(a) = b$ and $f \circ g = I$ (identity function). Then, $f'(b)$ is equal to

(a) 2 (b) 2/3 (c) ½ (d) none of these

26. Let $f:(0, \infty) \to R$ and $F(x) = \int^{x} f(t)dt$. If $F(x^2) = x^2(1 + x)$, then $f(4)$ equals

(a) 5/4 (b) 7/4 (c) 4 (d) 2

Level – II

27. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

(a) $-\frac{3b}{a^3}\text{cosec}^4\theta \cot^4\theta$ (b) $\frac{3b}{a^3}\text{cosec}^4\theta \cot\theta$ (c) $-\frac{3b}{a^3}\text{cosec}^4\theta \cot\theta$ (d) none of these

28. The derivative of $\sec^{-1}\left(\frac{1}{2x^2+1}\right)$ w.r.t. $\sqrt{1+3x}$ at $x = -\frac{1}{3}$

(a) does not exist (b) 0 (c) 1/2 (d) 1/3

29. Let f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h'(x) = [f(x)]^2 + [g(x)]^2$, $h(1) = 8$ and $h(0) = 2$, then $h(2) =$

(a) 1 (b) 2 (c) 3 (d) none of these

30. Let $f(t) = \ln(t)$. Then $\frac{d}{dx}\left(\int_{x^2}^{x^3} f(t)dt\right)$

(a) has a value 0 when $x = 0$
(b) has a value 0 when $x = 1$ and $x = 4/9$
(c) has a value $9e^2 - 4e$ when $x = e$
(d) has a differential coefficient $27e - 8$ for $x = e$

31. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ is equal to

(a) $P(x) + P''(x)$ (b) $P(x)$ (c) $P(x)\,P'''(x)$ (d) a constant

32. If $x = \int_0^y \frac{1}{\sqrt{1+4t^2}}dt$, then $\frac{d^2y}{dx^2}$ is

(a) 2y (b) 4y (c) 8y (d) 6y

33. Let $f(x)$ and $g(x)$ be two functions having finite non–zero. 3rd order derivatives $f'''(x)$ and $g'''(x)$ for all $x \in R$. If $f(x)\,g(x) = 1$ for all $x \in R$, then $\frac{f'''}{f'} - \frac{g'''}{g'}$ is equal to

(a) $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$ (b) $3\left(\frac{f''}{f} - \frac{g''}{g}\right)$ (c) $3\left(\frac{g''}{g} - \frac{f''}{g}\right)$ (d) $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$

34. If $F(x) = \frac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right)dt$, then $F'(4)$ equals

(a) $\frac{32}{9}$ (b) $\frac{64}{3}$ (c) $\frac{64}{9}$ (d) none of these

35. If $f(x) = \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$, then $f'(\theta)$ is

(a) 0 (b) -1 (c) independent of θ (d) none of these

36. If y 11. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(a) 1 (b) log 2 (c) – log 2 (d) – 1

37. If $f(x) = |\cos x - \sin x|$, then $f'(\pi/4)$ is equal to

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 0 (d) none of these

38. Let $f : (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1 - \sqrt{\frac{3}{2}}$ (B) $1 + \sqrt{\frac{3}{2}}$

(C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$

39. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0)$

(a) – 4 (b) 0 (c) – 2 (d) 4

40. If $x = e^{y + e^{y + e^{y \cdots\cdots\infty}}}$, $x > 0$ then $\frac{dy}{dx}$ is

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

41. Let f and g be real valued functions defined on interval (– 1, 1) such that $g''(x)$ is continuous, $g(0) \neq 0$. $g'(0) = 0$, $g''(0) \neq 0$ and $f(x) = g(x) \sin x$

Statement – 1 : $\lim_{x \to 0} \{g(x)\cot x - g(0)\operatorname{cosec} x\} = f''(0)$ and

Statement – 2 : $f'(0) = g(0)$

(a) Statement 1 is true, statement – 2 is true; Statement -2 is a correct explanation for Statement -1

(b) Statement 1 is True, Statement -2 is true; Statement -2 is NOT a correct explanation for Statement -1

(c) Statement – 1 is true, Statement – 2 is False

(d) Statement – 1 is False, Statement – 2 is True

42. **S1 :** if $f(x) = |x - 2|$, then $f'(f(x)) = 1$ for $x > 20$

S2 : if $f(x) = \frac{x}{1+|x|}$, then $f'(-1) = \frac{1}{4}$

S3 : If $f(0) = a$, $f'(0) = b$, $g(0) = 0$ and $(fog)'(o) = c$, then $g'(0) = c/b$

S4 : differential coefficient of $2\tan^{-1} x$ w.r.t $\sin^{-1}\frac{2x}{1+x^2}$ at $x = \frac{1}{2}$ is 1

(a) F T T T (b) T F T T (c) T T F F (d) T T T T

43. $\frac{d^2x}{dy^2}$ equals

(a) $\left(\frac{d^2x}{dy^2}\right)^{-1}$ (b) $-\left(\frac{d^2x}{dy^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

44. If $f(x) = x^n$, then the value of

$f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + \ldots\ldots\ldots + \frac{f^n(1)}{n!}$ is

(a) n (b) 2^n (c) 2^{n-1} (d) $\frac{n(n+1)}{2}$

45. If $\int_{\sin x}^{1} t^2 f(t)\,dt = 1 - \sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 3 (d) $\sqrt{3}$

46. If f(x) is differentiable and $\int_0^{t^2} x f(x)\,dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(a) 2/5 (b) – 5/2 (c) 1 (d) 5/2

Section II

Match the following

47. Observe the following list:

	List –I	**List –II**
(A)	Let $f(x) = \begin{cases} \tan^{-1}x, & \lvert x\rvert \geq 1 \\ \frac{x^2-1}{4}, & \lvert x\rvert < 1, \\ \frac{x^2-1}{4}, & \lvert x\rvert < 1 \end{cases}$ then f(x) is not differentiable at x equal to	P. –1
(B)	$f(x) = (x^2 - 4)\,\lvert x^2 - 5x + 6\rvert + \cos\lvert x\rvert$ is non derivable at x equal to	Q. 1
(C)	If $\sin(x + y) = e^{x+y} - 2$, then $\frac{dy}{dx}$ is equal to	R. 2
(D)	Let : R → R is defined by the equation $f(x + y) = f(x)\,f(y) \forall\ x, y \in R$, $f(0) \neq 0$ and $f'(0)=2$ then $\frac{f'(x)}{f(x)}$ is equal to	S. 3 T. None of the above values

48. **Column – I** **Column – II**

(a) $\lim_{x\to\frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$ is equal to (p) does not exist

(b) if $f(x) = \log_{x^2} (\log x)$, then $f'\left(\frac{1}{2}\right)$ is equal to (q) 0

(c) for the function $f(x) = \ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ if $\frac{dy}{dx} = \sec x + p$. then p is equal to (r) $-\frac{1}{\sqrt{2}}$

(d) $\lim_{x\to 0} \frac{1}{x} \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ is equal to (s) 4

Section III

Comprehension type

49. A function f: R$\to$ R satisfies the following conditions:
(1) $f(x) \neq 0$ for any $x \in R$ (2) $f(x + y) = f(x).\ f(y)$ for all x, y in R
(3) f(x) is differentiable (4) $f'(0) = 2$

1. The value of f(0) is
(a) 1 (b) -1 (c) 2 (d) 1 /2

2. The derivative of f(x), i.e., f′ satisfies the equation
(a) $f'(x + y) = f'(x) + f'(y)$ (b) $f'(x + y) = f'(x)\ f'(y)$
(c) $f'(x + y) = f'(x)\ f(y)$ (d) $f'(x + y) = f'(x) + f(y)$

3. The ratio $\frac{f'(x)}{f'(x)}$ for all x, equals to
(a) 1 (b) 2 (c) x (d) 2x

4. The function f(x) is
(a) e^x (b) e^{2x} (c) $\log |x|$ (d) 2^x

5. $\lim_{x \to 0} \frac{f(x) - f(-x)}{x} =$
(a) 1 (b) 2 (c) 3 (d) 4

Section IV

50. If $f_r(x)$, $g_r(x)$, $h_r(x)$; r = 1, 2, 3 are polynomials in x such that $f_r(a) = g_r(a) = h_r(a)$, r = 1, 2, 3

and $F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$ then $F'(x)$ at x = a is .....

51. The derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ with respect to $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$ is .....

52. If $f(x) = \sin (\log x)$ and $y = f\left(\frac{2x + 3}{3 - 2x}\right)$, then $\frac{dy}{dx}$ =.....

53. If $\phi(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ x & x^2 & x^3 \\ 0 & 2 & 6x \end{vmatrix}$ then $\phi'(1) =$ .....

54. If $x = e^t \cos t$, $y = e^t \sin t$ then $\frac{d^2y}{dx^2} =$ .....

55. If $f(x) = \log_x (\ln x)$, then $f'(x)$ at $x = e$ is .....

56. If $y = \cos 2x \cos 3x$, then y_n is equal to

(a) $6^n \cos\left(2x + \frac{n\pi}{2}\right)\cos\left(3x + \frac{n\pi}{2}\right)$ (b) $6^n \sin\left(2x + \frac{n\pi}{2}\right)\cos\left(\frac{3x + n\pi}{2}\right)$

(c) $\frac{1}{2}\left[5^n \sin\left(5x + \frac{n\pi}{2}\right) + \sin\left(x + \frac{\pi}{2}\right)\right]$ (d) none of these

57. If the function $f(x) = x^3 + e^x$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Subjective

1. Find the derivative of

$$f(x) = \begin{cases} \frac{x-1}{2x^2 - 7x + 5} & \text{when } x \neq 1 \\ -\frac{1}{3} & \text{when } x = 1 \end{cases}$$

at $x = 1$

2. Find the derivative with respect to x of the function

$(\log_{\cos x} \sin x)(\log_{\sin x} \cos x)^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $x = \frac{\pi}{4}$

3. Find $\frac{dy}{dx}$ at $x = 1$, when

$(\sin y)^{\sin\left(\frac{x}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x(\ln(x+2)) = 0$

4. If $y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1 + \sin^2\sqrt{\theta}}\, d\theta$, then find $\frac{dy}{dx}$ at $x = \pi$

Answers

1	C	8	D	15	D	22	A	29	D	36	D
2	C	9	B	16	B	23	B	30	B, C,D	37	D
3	B	10	17	17	A	24	B	31	C	38	AB
4	B	11	C	18	B	25	C	32	B	39	A
5	D	12	C	19	C	26.	C	33	B	40	C
6	D	13	D	20	D	27	C	34	A	41	A
7	A	14	A	21	A	28	A	35	B,C	42.	D
43.	D	44.	B	45.	C	46.	A				

47. A – P, Q B- S, C – Q, D – R, **48**. A $\to$ r, B $\to$ p, C $\to$ q, D $\to$ p,

49. 1 $\to$a, 2.$\to$c, 3$\to$b, 4$\to$ b, 5$\to$d

50. zero **51.** - 4 **52.** $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

53. –6 **54.** $\frac{1}{\sqrt{2}e^t}\sec^3\left(\frac{\pi}{4}+t\right)$ **55. -2/9**

56. D 57. 2

Subjective

2. $4-8\log_2 e+\frac{32}{16+\pi^2}$ 3. 0 4. 2π