Very Very Important Questions for XIIth Board Exam

[Mathematics]

1. If $A = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$ and $B' = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$, find $(A + B)'$. **1 mark**

2. If a and b are two vectors of magnitude 3 and 2/3 respectively such that a × b is a unit vector, write the angle between a and b. **1 mark**

3. Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$. **1 mark**

4. What is the principal value of $\cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \sin^{-1}\left(\sin\frac{3\pi}{4}\right)$? **1 mark**

5. Probabilities of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that the problem is solved. **1 mark**

6. If * is a binary operation on N defined as a * b = LCM of a and b, find (2 * 3) * 4. **1 mark**

7. Prove that the tangents to the curve $y = x^3 + 9$ at the points (–1, 5) and (1, –1) are parallel. **1 mark**

8. Find whether $y = \frac{a}{x} + b$ is a solution of $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$. **1 mark**

9. If a matrix has 9 elements, write all possible orders it can have. **1 mark**

10. Write the value of $\int_0^{\pi} \cos^5 x\, dx$. **1 mark**

11. Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)} dx$. **4 marks**

12. Show that $\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$ **4 marks**

Or

Solve for x, $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

13. If cos y = x cos (a + y), cos a ≠ 1, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ **4 marks**

Or

Evaluate $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx$.

14. Find the equation of the plane passing through the point (1, 1, – 1) and perpendicular to the planes **4 marks**

$$x + 2y + 3z - 7 = 0$$

and $2x - 3y + 4z = 0$

15. Discuss the continuity of the function **4 marks**

$$f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2}, & x \neq 0 \\ 2, & x = 0 \end{cases} \quad \text{At } x = 0$$

16. Using properties of determinants, Prove the following : **4 marks**

$$\begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2 + b^2 - c^2) \end{vmatrix}$$

$= abc\,(a^2 + b^2 + c^2)^3$

17. If the sum of two unit vectors a and b is a unit vector, show that the magnitude of their difference is $\sqrt{3}$. **4 marks**

18. Evaluate $\int_{-1}^{2} |x^3 - x|\,dx$ **4 marks**

OR

Evaluate $\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x}\,dx$

19. Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of aces.
Gambling is considered a social evil, explain. **4 marks**

20. Find the derivative of $\tan^{-1}\left[\frac{\sqrt{1+x^2} - 1}{x}\right]$ with respect to $\tan^{-1}\left[\frac{2x}{1+x^2}\right]$. **4 marks**

21. Find the equation of the plane passing through the point (3, – 3, 1) and perpendicular to the line joining the point (3, 4, –1) and (2, – 1, 5). Also, find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane. **4 marks**

OR

Find the distance of the point (3, 4, 5) from the plane x + y z = 2 measured parallel to the line $2x = y = z$

22. Solve the following differential equation **4 marks**

$$ye^{x/y}dx = (xe^{x/y} + y)dy$$

23. Find the particular solution of the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$. Give, that, $y = 0$ when $x = 0$. **6 marks**

24. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ **6 marks**

Find AB. Use the result to solve the following system of linear equations

$2x - y + z = -1;\ -x + 2y - z = 4;\ x - y + 2z = -3$

25. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die? **6 marks**

26. A toy manufacturer produces two types of toys; a basic version toy A and a deluxe version toy B. Each toy of type B takes twice as long to produce as one toy of type A. The company has time to make a maximum of 2000 toys of type A per day, the supply of plastic is sufficient to produce 1500 toys per day and each type requires equal amount of it. Type B requires a fency dress of which there are only 600 per day available. If the company makes a profit of Rs. 30 and Rs. 50 per toy, respectively on A and B, how many of each type should be produced per day in order to maximize profit? Make an LPP and solve it graphically. **6 marks**

27. In an activity organized in the school, Rohan was given the task to put the slogan 'Satyamev Jayte' on a trapezium shaped card sheet. If length of three sides of a trapezium other than base are equal 10 cm, then find the area of the trapezium when it is maximum. Explain the meaning of 'Satyamev Jayte'. **6 marks**

28. Evaluate $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$. **6 marks**

29. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $r = 2i - j + 2k + \lambda(3i + 4j + 2k)$ and the plane $r \cdot (i - j + k) = 5$. **6 marks**

Or

Evaluate $\int_{-1}^{1} e^x dx$ as a limit of sums.