Physics

DISHA CLASSES/JEE Main/Part Test-2

Q.1 (C) For the refraction at the point A $\frac{\sin i}{\sin 60} = \frac{1}{\mu}$

Also from the Brewster's law $\tan i = \frac{1}{\mu}$ [light is passing from denser to rarer]

So $\cos i = \frac{\sqrt{3}}{2} \Rightarrow i = 30^\circ$

So $\tan i = \frac{1}{\mu} = \frac{1}{\sqrt{3}} \Rightarrow \mu = \sqrt{3}$

Q.2. (a) After refraction from the liquid image will be formed at $\mu h = \mu \times 15$ cm, so distance of this image from the pole is $15\mu + 5$ → this should be equal to the radius of curvature of the mirror

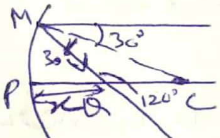
$15\mu + 5 = 25 \Rightarrow \mu = \frac{4}{3}$

Q.3.(b) Distance of the image of O, from the water surface (formed by the refraction only)

$x_1 = \left(\frac{H-d}{\mu}\right)$. The distance of the image of O formed after reflection from the mirror, then refraction from the water, from the water surface

$x_2 = \left(\frac{d+H}{\mu}\right)$. So $x_2 - x_1 = \frac{2d}{\mu}$

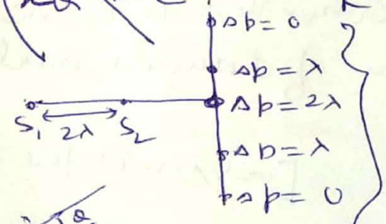
Q.4 (d)



Applying sine rule in Δ MQC

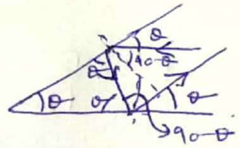
$\frac{\sin 30}{R-x} = \frac{\sin 120}{R} \Rightarrow x = R\left(1 - \frac{1}{\sqrt{3}}\right)$

Q.5. (C)



5 Maximas

6. (b)



$3\theta = 180 \Rightarrow \theta = 60^\circ$

7. (C) $\left\{\frac{hc}{\lambda/2} = W + E\right\} \times 2$

$\frac{\lambda c}{\lambda/3} = W + 2E$

$\frac{hc}{\lambda} = W$

8. (d.) From the Moseley equation

$\sqrt{f} = a\,(z_1 - b)$ $\quad z_1 = 31$ $\}$ for K_α. $b = 1 \Rightarrow f = \frac{25}{9}f$

$\sqrt{f'} = a\,(z_2 - b)$ $\quad z_2 = 50$

9. (a) As 10.2 eV energy is required for the electron to get excited from $n=1$ to $n=2$, with 9.5 eV no excitation will take place. So no emission spectra.

10. (b) $\frac{1}{x} = R\left(\frac{1}{1} - \frac{1}{\infty}\right)$ for the shortest wavelength of Lyman series.

$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right)$ for the 1st line of Balmer series.

$\Rightarrow \lambda = \frac{36}{5}x$

11. (C) There's no line in the Brackett series $\Rightarrow$ highest excited state is $n=4$. Hence in the Balmer series the two wave numbers $\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right)$ & $\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right)$ only will be present.

12. (a) The concentration should be in the reciprocal of ratio of decay constants as Rate of decay $= N\lambda$ So $N_1\lambda_1 = N_2\lambda_2$

13. (C) Initial activity $= N_0\lambda = \left(\frac{m}{M} \times N_A\right)\lambda$

14. (a) $\frac{N_0 e^{-11\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e^2} = e^{-2} \Rightarrow 10\lambda t = 2 \Rightarrow t = \frac{1}{5\lambda}$

15. (C) Look at the half lives. In 5 minutes most of the Rn & Po would have been converted into Pb but in 5 minutes very few atoms of Pb would have been converted into Bi.

16. (b) As mass Δm has been converted into energy. So the binding energy/nucleon of daughter nucleus must be greater.

17. (a) $\Delta m c^2 = \left\{\frac{1}{2}\frac{M}{2}v^2\right\} \times 2$ [as Δm is the loss of mass]

18. (C) $\frac{P\,dt}{4\pi r^2 c\,dt} = \frac{1}{2}\varepsilon_0 E^2 \Rightarrow \frac{P_1}{P_2} = \left(\frac{E}{E'}\right)^2 \Rightarrow E' = \frac{E}{\sqrt{2}}$

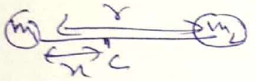
19. (C) The actual depth of the bottom $= h_1 + h_2$

the apparent depth $= \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$

so shift $= (h_1 + h_2) - \left\{ \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \right)$

20. (d) $L = I\omega \Rightarrow L^2 = I^2\omega^2 = \frac{1}{2} I\omega^2 \times 2I$

$\Rightarrow KE = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \times \frac{1}{2I} = n^2\hbar^2 \times \frac{(m_1+m_2)}{2m_1m_2r^2}$



$x = \left(\frac{m_2 r}{m_1+m_2}\right)$ & $r - x = \left(\frac{m_1 r}{m_1+m_2}\right)$ } so $I = m_1x^2 + m_2(r-x)^2$

$I = \frac{m_1 m_2 r^2}{(m_1+m_2)}$

Q.21. (48°) $\delta = (i+e) - (r_1 + r_2) = i + e - A$

$10 = (20+38) - A \Rightarrow A = 48^\circ$

[δ is same for two i, 20° & $38^\circ \rightarrow$ if $i = 20^\circ$ $e = 38^\circ$]

Q.22. 110 cm The focal length of the lens $\frac{1}{f} = (1.5-1)\left(\frac{1}{10} - \frac{1}{20}\right)$

$\Rightarrow f = 40$ cm.

The final image will be formed at the same point where the object is, if image of the lens is at the center of curvature of the mirror.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get $v = 80$ cm so $d = 80 + 30 = 110$ cm

Q.23. (28 mm) The position of min. for two wavelengths will coincide

$\frac{D}{d}(n_1 - \frac{1}{2})\lambda_1 = \frac{D}{d}(n_2 - \frac{1}{2})\lambda_2$ for 1st time $n_1 = 4$ & $n_2 = 3$

Similarly get the two sides equal for n_1 & n_2

then Get $x = \frac{D(n_1 - \frac{1}{2})\lambda_1}{d}$ [put the 2nd value of n_1

Q.24. (2) $0.528 \times 10^{-10} \frac{n^2}{2} = 0.528 \times 10^{-10} \Rightarrow n = 2$

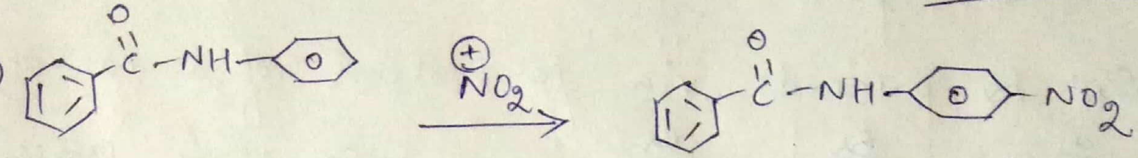
[here $Z = 4$]

Q.25. 2 minutes $\frac{1}{3} = e^{-\lambda t_2}$ & $\frac{2}{3} = e^{-\lambda t_1}$

$\Rightarrow e^{-\lambda(t_2 - t_1)} = \frac{1}{2}$ taking log.

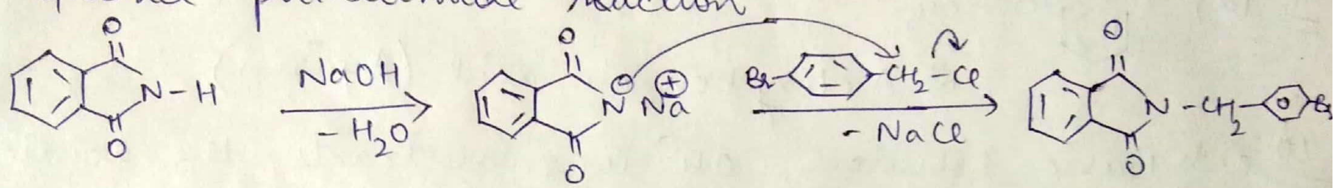
TS-20 / JEE Main / Part Test - 2 CHEMISTRY

1 (d)

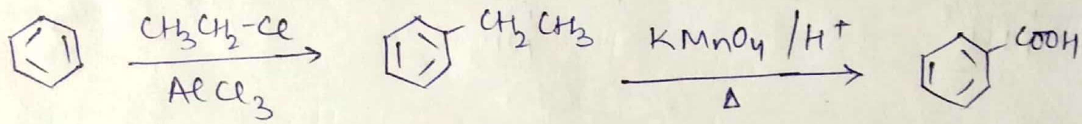


-NH group is ring activating while $-\overset{O}{\overset{\|}{C}}-$ group is ring deactivating. So. $E^{\oplus}$ substitution occurs on more activated benzene ring.

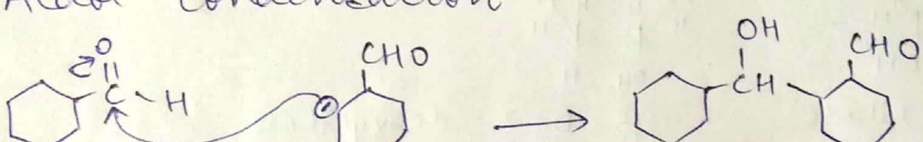
2 (c) Gabriel phthalimide reaction



3 (a)

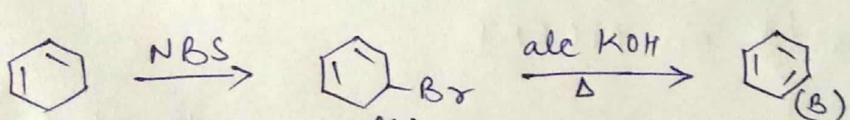


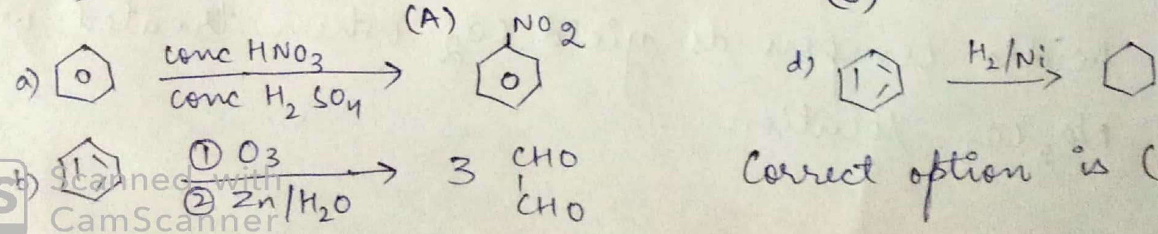
4 (b) Aldol Condensation



5 (b) Electron releasing groups increase the rate of E.S.R while $e^{\ominus}$ withdrawing groups decrease the rate

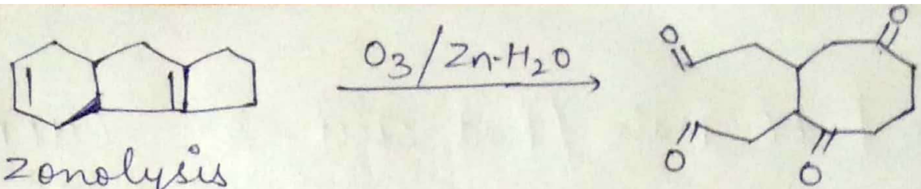
6 (c)





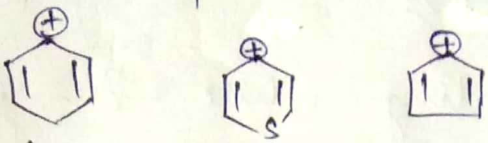
Correct option is (C)

7 (b)



Ozonolysis

8 (c) S_N' rx proceeds via formation of a carbocation



(A) Non aromatic

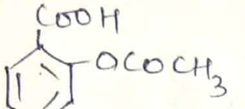
(B) Aromatic

(C) Anti aromatic

Stability order

Aromatic > Non aromatic > Anti aromatic

9 (d)

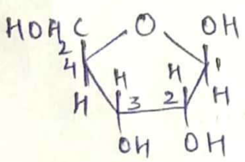


2-Acetoxy benzoic acid (Aspirin)

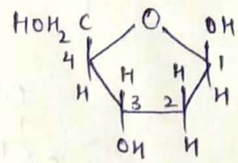
10 (c) In Basic solution, $OH^{\ominus}$ ions abstracts the acidic H^+ ion

$$R-\underset{NH_2}{\underset{|}{CH}}-COOH \xrightarrow{\overset{\ominus}{O}H} R-\underset{NH_2}{\underset{|}{CH}}-COO^{\ominus} + H_2O$$

11 (b)



β-D-ribose



β-D-2-deoxyribose

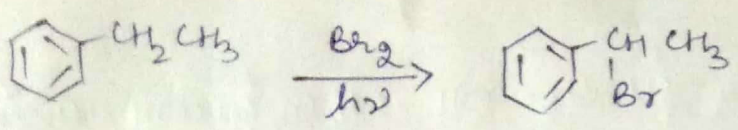
12 (b) Based on NCERT; fact

13 (b) Nylon-66 $\left[-NH-(CH_2)_6-NH-CO-(CH_2)_4-\overset{O}{\overset{||}{C}}-\right]_n$

14 (d) Fact; Based on NCERT

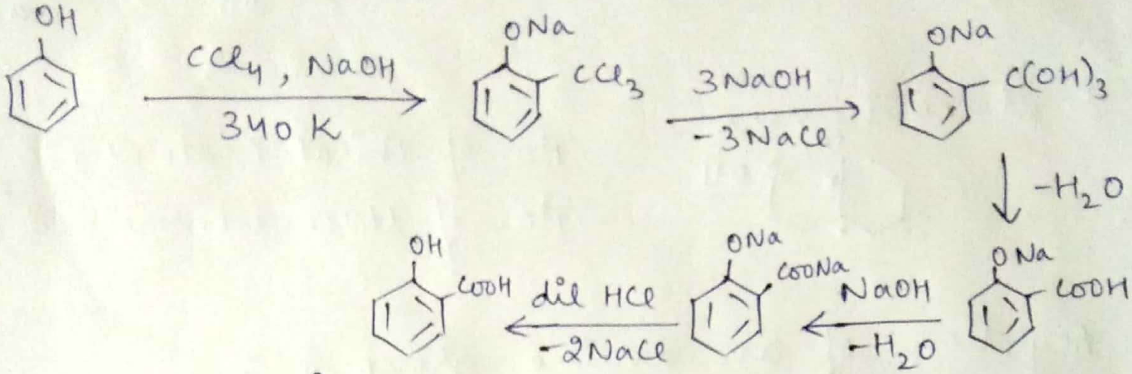
15 (b) Acidic compounds yield CO_2 when treated with Na_2CO_3 solution

16 (c)



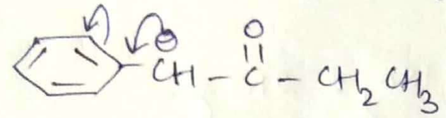
(Allylic bromination)

17 (c)



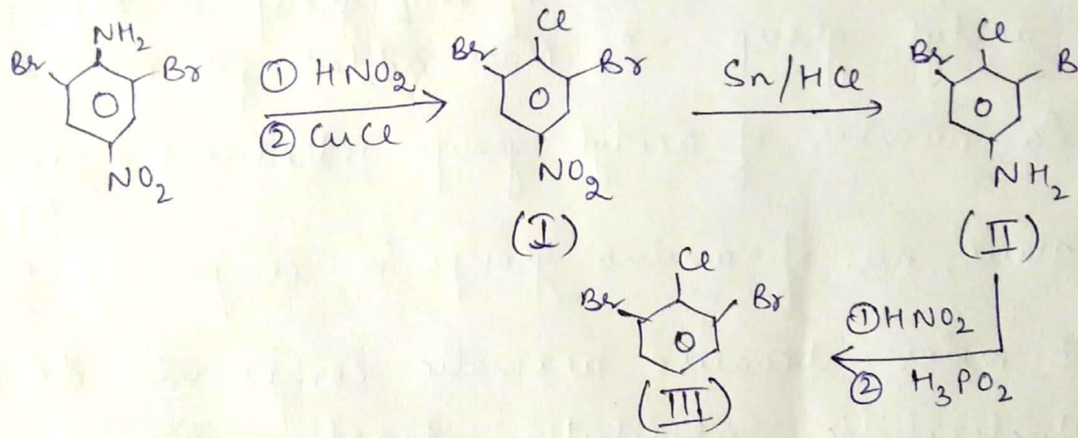
Salicylic acid
(O-Hydroxybenzoic acid)

18 (c)



Resonance stabilisation because of more resonating structures

19 (c)



20 (b) Basic strength: Amines > Ammonia
& 2° amine > 1° amine > 3° amine
(when R = $-CH_3$)

21 3° alcohols reacts fastest with Lucas reagent

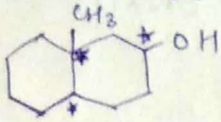
$\therefore\ x = 3$

$$\frac{2x}{5} = \frac{2(3)}{5} = 1.2 \text{ Ans}$$

22 4 isomers

$CH_3-\overset{*}{C}H(CH_2OH)-COOH$ $\quad$ $CH_3-\overset{*}{C}H(OH)-CH_2-COOH$ $\quad$ $CH_3CH_2\overset{*}{C}H(OH)COOH$ $\quad$ $CH_3\overset{*}{C}H(OCH_3)COOH$

23 8 stereo-isomers



No. of chiral C-atoms = 3

No. of stereo-isomers = $2^3 = 8$

24 Acylation of an amine group is :

$RNH_2 + CH_3COCl \longrightarrow RNHCOCH_3 + HCl$

Mass of product = 390 g

Mass of amine = 180 g

Increase in mass = 390g − 180g = 210 g

Mass of one molecule of CH_3CO- group = 43 g

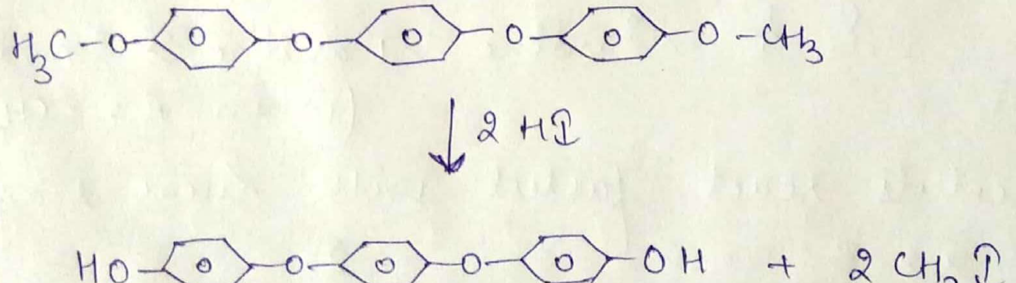
Since, one H-atom of NH_2 group is replaced by acetyl group ∴ Mass added per molecule = (43−1)

= 42 g

So, number of acetyl groups introduced = $\frac{210}{42} = 5$

Thus, no. of amino groups in compound = 5 Ans

25 2 moles because aromatic compounds donot give Nucleophilic Substitution reaction easily



Mathematics

Mathematics

Q1(C) Plane can be rewritten as

$$x + 3y + z - 4 + \lambda(2x - y) = 0$$

which is intersection of plane $x + 3y + z - 4 = 0$ & $2x - y = 0$

so line is line of intersection of two planes

$$\bar{b}_1 = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 7\hat{k}$$

Any point which lies on both plane, lies on line.

In alternative (C) $\bar{b}$ is same & line passes through $(0, 0, 4)$ which lies on both planes

Q2

$$\int \frac{\left(p x^{p+2q-1} - q x^{q-1}\right)}{\left(x^{p+q} + 1\right)^2} dx = \int \frac{p x^{p+2q-1} - q x^{q-1}}{(x^q)^2 \left(x^p + x^{-q}\right)^2} dx$$

$$= \int \frac{p \frac{x^{p+2q-1}}{x^{2q}} - q \frac{x^{q-1}}{x^{2q}}}{\left(x^p + x^{-q}\right)^2} dx$$

$$= \int \frac{p x^{p-1} - q\left(x^{-q-1}\right)}{\left(x^p + x^{-q}\right)^2} dx$$

Now sub. $x^p + x^{-q} = t$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{x^p + x^{-q}} = -\frac{x^q}{x^{p+q} + 1}$$

Q3 $S_n = \sum_{r=1}^{n} \frac{1}{r+\sqrt{rn}} = \sum_{r=1}^{n} \frac{1/n}{r/n + \sqrt{\frac{rn}{n}}}$

$= \sum_{r=1}^{n} \frac{1/n}{r/n + \sqrt{r/n}}$

$= \int_0^1 \frac{dx}{x+\sqrt{x}} = \int_0^1 \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

Sub. $\sqrt{x}+1 = t$

Q4 $\int e^x (\tan x - x - 2\tan x \sec^2 x)\,dx$

$\int e^x(\tan x + \sec^2 x - \sec^2 x - x - 2\tan x \sec^2 x)\,dx$

$\int e^x(\tan x + \sec^2 x)dx - \int (1+\tan^2 x + x + 2\tan x\sec^2 x)\, e^x dx$

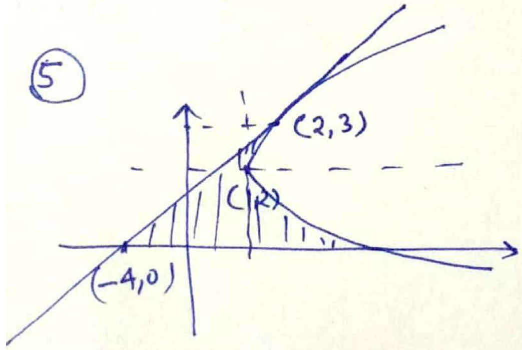
$e^x \tan x - \int (1+x)e^x dx - \int (\tan^2 x + 2\tan x \sec^2 x) e^x dx$

$e^x \tan x - x e^x - (\tan^2 x)\, e^x + c$

so $f(x) = \tan x - x - \tan^2 x$

$f(\pi/4) = -\pi/4$

⑤



Desired area is shaded region

$(y-2)^2 = x-1$

$\Rightarrow 2(y-2)\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right|_{2,3} = \frac{1}{2}$

Eqn of tangent $y-3 = \frac{1}{2}(x-2)$

$\Rightarrow x = 2y-4$

Shaded area $= \int_0^3 (x_{parabola} - x_{Tangent})\,dy$

(6) differentiating both sides with respect to x

$\Rightarrow xy = 2x + y'$

$\Rightarrow \frac{dy}{dx} - xy = -2x$

$I.F. = e^{\int -x\,dx} = e^{-x^2/2}$

$y \cdot e^{-x^2/2} = \int -2x\, e^{-x^2/2}\, dx$

$y\, e^{-x^2/2} = +2 e^{-x^2/2} + c$

$y = 2 + c \cdot e^{x^2/2}$ or $y = 2 - c\, e^{x^2/2}$

(7) Since both lines passes through $(1,-1,3)$ so their point of intersection is $(1,-1,3)$

so plane passes through $(1,-1,3)$ & greatest distance from the origin $\Rightarrow$ line joining origin & $(1,-1,3)$ is normal to plane

$\Rightarrow \bar{n} = \hat{i} - \hat{j} + 3\hat{k}$

equation of plane $\bar{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = (\hat{i} - \hat{j} + 3\hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k})$

$\bar{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 11$

$\Rightarrow x - y + 3z = 11$

(8) $\bar{a} \times \bar{b} = \hat{j} - \hat{k}$

Taking cross product both sides by a

$\bar{a} \times (\bar{a} \times \bar{b}) = \bar{a} \times (\hat{j} - \hat{k})$

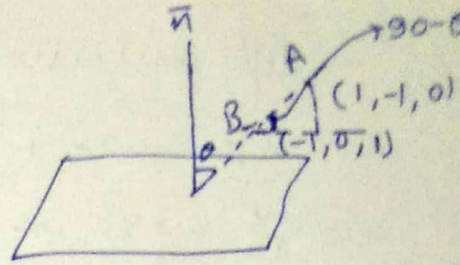
$(\bar{a} \cdot \bar{b})\bar{a} - |a|^2 \bar{b} = \bar{a} \times (\hat{j} - \hat{k})$

$1(\bar{a}) - |\bar{a}|^2 \bar{b} = \bar{a} \times (\hat{j} - \hat{k})$

$\Rightarrow \bar{b} = \dfrac{\bar{a} - \bar{a} \times (\hat{j} - \hat{k})}{|\bar{a}|^2}$

Now subsitute $\bar{a}$.

(9) Suppose Angle between $\overline{AB}$ & $\overline{n}$ is θ then projection of $\overline{AB}$ on plane is $|AB| \sin\theta$



$$\overline{AB} = -2\hat{i} + \hat{j} + \hat{k}$$

$$\overline{n} = 2\hat{i} + \hat{j} + 6\hat{k}$$

$$\cos\theta = \frac{-4+1+6}{\sqrt{6}\sqrt{41}} = \frac{3}{\sqrt{6}\sqrt{41}} = \frac{\sqrt{3}}{\sqrt{82}}$$

$$\sin\theta = \cancel{\frac{\sqrt{55}}{\sqrt{82}}} \quad \frac{\sqrt{79}}{\sqrt{82}}$$

$$\text{Projection} = \cancel{\frac{\sqrt{6}\cdot\sqrt{55}}{\sqrt{82}}} = \cancel{\frac{\sqrt{165}}{\sqrt{41}}}$$

$$= \frac{\sqrt{6}\sqrt{79}}{\sqrt{82}} = \frac{\sqrt{237}}{\sqrt{41}}$$

(10)

$$\bar{a} + \bar{b} + \bar{c} = \bar{x}$$

Taking dot product with $\bar{x}$

$$\Rightarrow \bar{a}\cdot\bar{x} + \bar{b}\cdot\bar{x} + \bar{c}\cdot\bar{x} = \bar{x}\cdot\bar{x}$$

$$1 + 3/2 + |c||x|\cdot\cos\theta = 2^2$$

$$\Rightarrow |c||x|\cos\theta = 3/2 \Rightarrow \cos\theta = \frac{3/2}{1\times 2}$$

$$= 3/4$$

(11)

(11) $$I = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin(x^2)\, dx}{\sin(x^2) + \sin(\ln 6 - x^2)}$$

Sub. $x^2 = t$

$$I = \int_{\ln 2}^{\ln 3} \frac{\sin t \left(\frac{dt}{2}\right)}{\sin t + \sin(\ln 6 - t)}$$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t \, dt}{\sin t + \sin(\ln 6 - t)} \quad -(1)$$

replacing t by $\ln 3 + \ln 2 - x = \ln 6 - t$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)\, dt}{\sin(\ln 6 - t) + \sin t} \quad -(2)$$

(1) + (2)

$$\Rightarrow 2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} dt = \frac{1}{2}\ln 3/2$$

$$\Rightarrow I = \frac{1}{4}\ln 3/2$$

(12) $$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx + \int_1^2 (1+\cos^8 x)(ax^2+bx+c)dx$$

$$\Rightarrow \int_1^2 (1+\cos^8 x)(ax^2+bx+c)\,dx = 0$$

Above integration is zero $\Rightarrow$ Integrand is both +ive & -ive in $x \in [1,2]$ $\Rightarrow$ ax^2+bx+c is both +ive & -ive in $x \in [1,2]$ so $ax^2+bx+c = 0$ at least once in $x \in [1,2]$

(13) $\int f^{-1}(x)\,dx$ sub. $x = f(t)$

$dx = f'(t)\,dt$

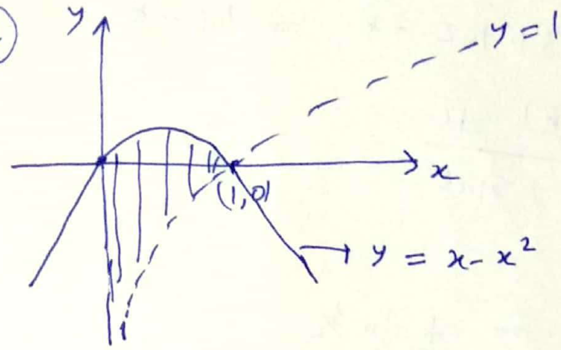
$\int t\, f'(t)\,dt$

$= t\,f(t) - \int 1 \times f(t)\,dt$

$= [f^{-1}(x)]\, x - g(t) + c$

$= x\, f^{-1}(x) - g\{f^{-1}(x)\} + c$

(14)



Shaded area

$= \int_0^1 (x - x^2 - \ln x)\,dx$

$= \frac{x^2}{2} - \frac{x^3}{3} - (x\ln x - x)\Big|_0^1$

$= \frac{1}{2} - \frac{1}{3} + 1 - 0 - \lim_{x\to 0}(x \ln x)$

$= \frac{3}{2} - \frac{1}{3} + 0$

$= \frac{7}{6}$

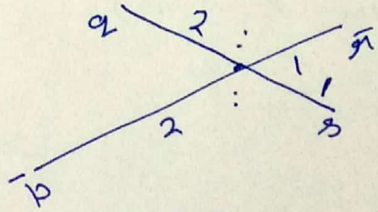
(15) $\bar{p} + 2\bar{r} = \bar{q} + 2\bar{s}$

$\Rightarrow \frac{\bar{p} + 2\bar{r}}{3} = \frac{\bar{q} + 2\bar{s}}{3}$

Using section formulae

$\Rightarrow$ point which divides $\bar{p}, \bar{r}$ in $2:1$ is same as that of point which divides $\bar{q}, \bar{s}$ in $2:1$

$\Rightarrow$ PR & QS trisects each other.



(16) Dividing by Tany siny on both sides

$\Rightarrow \cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{1}{x} \operatorname{cosec} y = \frac{1}{x^2}$

$\Rightarrow -\frac{d}{dx}(\operatorname{cosec} y) + \frac{1}{x} \operatorname{cosec} y = 1/x^2$

$\Rightarrow \frac{d}{dx}(\operatorname{cosec} y) - \frac{1}{x} \operatorname{cosec} y = -1/x^2$

$I.F. = e^{\int -1/x \, dx} = e^{-\ln x} = 1/x$

$(\operatorname{cosec} y)(\frac{1}{x}) = \int (-1/x^2)(1/x)\, dx + c$

$(\operatorname{cosec} y)(1/x) = \frac{1}{2x^2} + c$

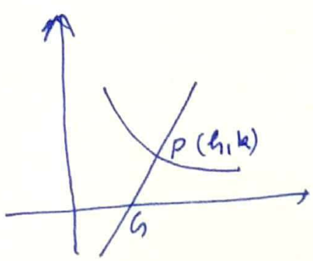
$\Rightarrow 1 = \frac{1}{2x} \sin y + xc \sin y$

$\Rightarrow 2x = \sin y + 2c x^2 \sin y$

$= \sin y (1 + 2c x^2)$

$2x = \sin y (1 + c' x^2)$

(17)



let P be (h,k)

Eqn of normal at P is

$y - k = -\frac{1}{\frac{dy}{dx}\big|_{h,k}} (x - h)$

For x intercep (for G)

$y = 0$

$\Rightarrow -k = -\frac{1}{\frac{dy}{dx}\big|_{h,k}} (x - h)$

$\Rightarrow x = h + k \frac{dy}{dx}\big|_{h,k}$

$\Rightarrow G = (h + k\frac{dy}{dx}\big|_{h,k}, 0)$

$\Rightarrow$ Given condition

$|h + k\frac{dy}{dx}\big|_{h,k}| = |2h|$

$\Rightarrow \frac{dy}{dx}\big|_{h,k} = \pm\frac{h}{k} \Rightarrow \frac{dy}{dx} = \pm\frac{x}{y}$

$\Rightarrow y\,dy = \pm(x\,dx)$

$\Rightarrow y^2/2 = \pm(x^2/2 + c) \Rightarrow$

Taking + sign

$\frac{y^2}{2} = \frac{x^2}{2} + c$ (hyperbola)

~~Single~~ Taking -ive sign

$\frac{y^2}{2} = -\left(\frac{x^2}{2} + c\right) \Rightarrow$ ellipse.

(18) Equations are

$x + y - 2z = -1 \quad - (1)$

$-2x + y + z = -1 \quad - (2)$

$x - 2y + z = 2 \quad - (3)$

$$\Delta = \begin{vmatrix} 1 & 1 & -2 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 0$$

& $(1) + (3) = -(2)$

$\Rightarrow$ these three ~~planes have~~ equation have infinite solutions.

Let x be x_0

From (1) & (2) $\Rightarrow$ $x_0 + y - 2z = -1 \quad - (1)$

$-2x_0 + y + z = -1 \quad - (2)$

$(1) - (2) \Rightarrow 3x_0 - 3z = 0 \Rightarrow z = x_0$

sub. x & z in (1)

$\Rightarrow x_0 + y - 2x_0 = -1$

$\Rightarrow y = x_0 - 1$

so (x_0, y_0, z_0) is in the form $(x_0, x_0 - 1, x_0)$

$$\frac{z_0^2 - y_0^2 + 1}{x_0} = \frac{x_0^2 - (x_0 - 1)^2 + 1}{x_0} = 2$$

(19) $I = \int_0^{\pi} \frac{x^2 \sin^4 x \, dx}{x^2 + \pi^2 - 2\pi x + x^2} = \int_0^{\pi} \frac{x^2 \sin^4 x}{(x-\pi)^2 + x^2} dx$ — ①

replace x by $\pi - x$

$I = \int_0^{\pi} \frac{(\pi - x)^2 \sin^4 x}{x^2 + (\pi - x)^2} dx$ — ②

① + ②

$\Rightarrow 2I = \int_0^{\pi} \sin^4 x \, dx$

$= \int_0^{\pi} \left(\frac{1 - \cos 2x}{2}\right)^2 dx = \frac{1}{4} \int_0^{\pi} (1 - 2\cos 2x + \cos^2 2x) dx$

$I = \frac{1}{8} \int_0^{\pi} \left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right) dx$

$= \frac{3\pi}{16}$

(20) $\int \frac{(x^4 - 4) dx}{x^2 \cdot \sqrt{x^2(4/x^2 + 1 + x^2)}} = \int \frac{(x^4 - 4) dx}{x^3 \sqrt{4/x^2 + 1 + x^2}}$

$= \int \frac{(x - 4/x^3) dx}{\sqrt{4/x^2 + 1 + 2x}}$

Sub. $4/x^2 + 1 + 2x = t$

$\Rightarrow (-8/x^3 + 2) dx = dt$

$\Rightarrow (1 - 4/x^3) dx = dt/2$

so given integral $= \int \frac{dt/2}{\sqrt{t}} = \sqrt{t}$

$= \sqrt{4/x^2 + 1 + 2x}$

(21) let angle between $\bar{u}$ & $\bar{v}$ be θ

$\Rightarrow |\bar{u} \times \bar{v}|^2 = \left|\frac{\bar{u} - \bar{v}}{2}\right|^2$

$\Rightarrow \sin^2\theta = \frac{1}{4}\left[|u|^2 + |v|^2 - 2\bar{u}\cdot\bar{v} \right]$

$\Rightarrow 4\sin^2\theta = 1 + 1 - 2\cos\theta$

$\Rightarrow 4 - 4\cos^2\theta = 2 - 2\cos\theta$

$4\cos^2\theta - 2\cos\theta - 2 = 0 \Rightarrow 2\cos^2\theta - \cos\theta - 1 = 0$

$\Rightarrow (2\cos\theta + 1)(\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = -\frac{1}{2}, 1$

Vectors are non co-linear $\Rightarrow \cos\theta = -\frac{1}{2}$

$|\bar{u} + \bar{v}| = \sqrt{|u|^2 + |v|^2 + 2\bar{u}\cdot\bar{v}}$

$= \sqrt{1 + 1 + 2(-\frac{1}{2})} = \cancel{\sqrt{3}}\ 1$

(22) $\int dP = \int (100 - 12\sqrt{x})\, dx$

$P = 100x - \frac{2 \times 12 \cdot x^{3/2}}{3} + c$

$P = 100x - 8x^{3/2} + c$

~~let when no of workers = x_0 then P = 2000~~

~~$\Rightarrow 2000 =$~~

when no. of additional workes $(x) = 0 \Rightarrow P = 2000$

$2000 = 0 + c \Rightarrow c = 2000$

$P = 100x - 8 \cdot x^{3/2} + 2000$

$P(25) = \cancel{1000}\ 100 \times 25 - 8 \cdot 25^{3/2} + 2000$

$= 2500 + 2000 - 1000 = 3500$

(23) let the plane be $Ax + By + Cz + D = 0$

Algebraic sum of perpendicular distances.

$$= \frac{A+2B+4C+D}{\sqrt{A^2+B^2+C^2}} + \frac{-3A+5B+2C+D}{\sqrt{A^2+B^2+C^2}} + \frac{(-A+2B+3C+D)}{\sqrt{A^2+B^2+C^2}} = 0$$

$$\Rightarrow -3A + 9B + 9C + 3D = 0$$

$$\Rightarrow -A + 3B + 3C + D = 0 \quad -(1)$$

From (1) it can be said that plane $Ax + By + Cz + D = 0$ passes through $(-1, 3, 3) = (a, b, c)$

(24) $\int_0^{\pi/2} [f(2x) + f''(2x)] \sin 2x \, dx$

Sub. $2x = t$

$$\int_0^{\pi} (f(t) + f''(t)) \sin t \frac{dt}{2} =$$

$$= \frac{1}{2}\int_0^{\pi} f(t)\sin t \, dt + \frac{1}{2}\int_0^{\pi} f''(t)\sin t \, dt$$ $\longrightarrow$ integration by parts

$$= \frac{1}{2}\int_0^{\pi} f(t)\sin t \, dt + \frac{1}{2}\left[f'(t)\sin t\Big|_0^{\pi} - \int_0^{\pi} f'(t)\cos t \, dt \right]$$

$$= \frac{1}{2}\int_0^{\pi} f(t)\sin t \, dt + \frac{1}{2}\left[0 - 0 - \left(f(t)\cos t\right)\Big|_0^{\pi} + \int_0^{\pi} f(t)(-\sin t)\,dt\right]$$

$$= \cancel{\frac{1}{2}\int_0^{\pi} f(t)\sin t \, dt} - \frac{1}{2}[-f(\pi) - f(0)] - \cancel{\frac{1}{2}\int_0^{\pi} f(t)\sin t \, dt}$$

$$= \frac{1}{2}[f(\pi) + f(0)] = 7$$

$$\Rightarrow 3 + f(0) = 14 \Rightarrow f(0) = 11$$

25

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \ln x + 2$$

$$I.F. = e^{\int \frac{1}{x\ln x}dx} = e^{\ln \ln x} = \ln x$$

Solution is $y(\ln x) = \int (\ln x)(\ln x + 2)\,dx + c$

$$y(\ln x) = \int[(\ln x)^2 + 2\ln x]dx + c$$

Sub. $\ln x = t \Rightarrow x = e^t$

$$= \int (t^2 + 2t)\,e^t\,dt + c$$

$$= t^2 e^t + c$$

$$= (\ln x)^2 e^{\ln x} + c$$

$$y(\ln x) = x(\ln x)^2 + c$$

when $x = 1 \Rightarrow y = 0$

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$$y(\ln x) = x(\ln x)^2$$

$$\Rightarrow y = x(\ln x)$$

$$y'(e) = 1 + \ln x\Big|_{x=e} = 2$$

$$y(e) = e\ln e = e$$

$$y(e) - y'(e) = e - 2$$

$[y(e) - y'(e)] = 0$ because $e - 2 \approx 0.41$