Maths Solutions

(61)

For function to be Onto

(i) 3 elements of A have same image & other 2 have different images

No. of ways $= {}^5C_3 \times 3! = 60$

(ii) Two elements have same image, other two have same image & one remaining element has different image

No. of ways $= \left(\dfrac{{}^5C_2 \times {}^3C_2}{2!}\right) \times 3! = 90$

Division of 5 objects in a group of 2, 2, 1

So Total $= 60 + 90 = 150$

$2x - x^2 > 0$ & $\log(2x - x^2) \geq 0$

& $2x - x^2 \geq 1$

$\Rightarrow 2x - x^2 > 0$. $2x - x^2 \geq 1 \Rightarrow x^2 - 2x + 1 \leq 0$

$\Rightarrow$ $\Rightarrow (x-1)^2 \leq 0 \Rightarrow x = 1$

$f(1) = f(2) = f(3) = 0 \Rightarrow$ Many one

$f(x)$ is continuous as $x \to -\infty \Rightarrow f(x) \to -\infty$

as $x \to \infty \Rightarrow f(x) \to \infty$

so Range $= (-\infty, 0)$

$x^2 - [x]^2 \geq 0 \Rightarrow (x - [x])(x + [x]) \geq 0$

$x - [x] \geq 0$ & $x - [x] = 0 \;\forall\; x \in I$

so domain contains all integers

& also those x for which $x + [x] \geq 0$

$\Rightarrow x \geq 0$

so domain $= [0, \infty) \cup \{I\}$

(65) $f(x) = \frac{x^2-x}{x^2+2x} = \frac{x-1}{x+2} = y$

$\Rightarrow x-1 = xy+2y$

$\Rightarrow x = \frac{2y+1}{1-y} = f^{-1}(y)$

$f^{-1}(x) = \frac{2x+1}{1-x}$

$\frac{d}{dx} f^{-1}(x) = \frac{d}{dx}\left(\frac{2x+1}{1-x}\right) = \frac{(1-x)2-(2x+1)(-1)}{(1-x)^2}$

$= \frac{3}{(1-x)^2}$.

(66) Every even natural no have image –ive integer
& every odd natural no have have image +ive integer
so one to one & Onto.

(67) $f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 0, & 1 \le x < \sqrt{2} \\ 1, & \sqrt{2} \le x < \sqrt{3} \\ 2, & \sqrt{3} \le x < 2 \\ 0 & x = 2 \end{cases}$ so Range $= \{0, 1,$

(68) $\frac{1-|x|}{2-|x|} \ge 0 \Rightarrow \frac{|x|-1}{|x|-2} \ge 0$ ~~$\Rightarrow |x| \in (2,\infty) \cup$~~

x 0 + + + 1 – – – 2 + + + $\Rightarrow |x| \in [0,1] \cup (2$

$\Rightarrow x \in [-1,1] \cup (2,\infty) \cup (-\infty,$

(69) $16-x \ge 2x-1 \Rightarrow 3x \le 17$, $x \le 17/3$ & $20-3x \ge 4x-5$

$\Rightarrow 25 \ge 7x$

$\Rightarrow x \le 3$

x should be integer & $4x-5 \ge 0$

$\Rightarrow x \ge 5/4$

so $x = 2, 3$

Domain $\{2,3\}$

(70) $n(A \times B) = 3 \times 4 = 12$

No. of relations from A to B $= 2^{12}$

(71) (i) if $a > 0$ then for $x < 0$ elements do not have image.

(ii) $y = |x|$ is function. each & every x have one & only one image.

(iii) $x > 1$ do not have image.

(iv) $0 < x < 1$ does not have image.

(72) (a) $f + g = x^2 + 4 + \frac{1}{\sqrt{x-1}} \Rightarrow D_{f+g} = (1, \infty)$

(b) $R_f = [4, \infty)$ & $R_g = (0, \infty]$

$R_f \cap R_g = [4, \infty)$

(73) Since $\ln x \in R \Rightarrow \tan(\ln x) \in R$ so Range $(-\infty, \infty)$

(74) let $\alpha = \frac{x^2+1}{x^2+2}$

$\Rightarrow \alpha x^2 + 2\alpha = x^2 + 1$

$\Rightarrow x^2 = \frac{1-2\alpha}{\alpha - 1} \geq 0 \Rightarrow \frac{1}{2} \leq \alpha < 1$

so $\sin^{-1}\alpha \in [\pi/6, \pi/2)$

(75) $f(-x) = -f(x)$

$\Rightarrow -\sin x + [x^2/a] = -(\sin x + [x^2/a])$

$\Rightarrow [x^2/a] = 0$

Since $x \in [-10, 10]$ so $\frac{x^2}{a} \in [0, \frac{100}{a}]$

$[x^2/a] = 0 \Rightarrow 0 < \frac{100}{a} < 1 \Rightarrow a > 100$

(76) $f''(x) - 2f'(x) - 15f(x) = 0$

$\Rightarrow a^2e^{ax} + b^2e^{bx} - 2(ae^{ax} + be^{bx}) - 15(e^{ax} + e^{bx}) = 0$

$\Rightarrow (a^2 - 2a - 15)e^{ax} + (b^2 - 2b - 15)e^{bx} = 0 \quad \forall x \in R$

$\Rightarrow a^2 - 2a - 15 = 0$ & $b^2 - 2b - 15 = 0$

(77) $x^{\frac{l+m}{(m-n)(n-l)}} + x^{\frac{m+n}{(n-l)(l-m)}} + x^{\frac{n+l}{(l-m)(m-n)}}$

$\Rightarrow x^{\frac{l^2-m^2+m^2-n^2+n^2-l^2}{(m-n)(n-l)(l-m)}} = x^0 = 1$

so derivative of given function $= 0$

(78) $\sin x = \pi/2 - 1 \Rightarrow x \in (0, \pi/2)$

(79) let $y = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow (y-1)x^2 + (y-1)x + y-2 = 0$

$\Rightarrow (y-1)^2 - 4(y-1)(y-2) \geq 0$

$\Rightarrow y^2+1-2y-4y^2+12y-8 \geq 0$

$\Rightarrow -3y^2+10y-7 \geq 0$

$\Rightarrow 3y^2-10y+7 \leq 0$

$\Rightarrow 3y^2-3y-7y+7 \leq 0 \Rightarrow 3y(y-1)-7(y-1) \leq 0$

$\Rightarrow (y-1)(3y-7) \leq 0$

$\Rightarrow 1 \leq y \leq 7/3$

Since in the numerator & denominator coeff of x^2 & x are same so $y \neq 1$

Range $(1, 7/3]$

(80) (a) $f(-x) = \frac{a^{-x}+1}{a^{-x}-1} = \frac{1/a^x+1}{1/a^x-1} = -\left(\frac{a^x+1}{a^x-1}\right) = -f(x)$

(b) $f(-x) = (-x)\left(\frac{a^{-x}+1}{a^{-x}-1}\right) = -x\left\{-\left(\frac{a^x+1}{a^x-1}\right)\right\}$

$= x\left(\frac{a^x+1}{a^x-1}\right) = f(x)$

(c) is odd (d) odd

(81) $f(2\pi+x) = f(x)$ so period is 2π

(82) $(a,a) \in R$ because $a-a=0 \in I$

$\Rightarrow$ Reflexive relation

if $(x,y) \in R \Rightarrow x-y = I_1$

$(y,z) \in R \Rightarrow y-z = I_2$

so $(x,y) \in R \Rightarrow R$ is transitive

~~$(x,y) \in R$~~ $(x,y) \in R \Rightarrow x-y$ is integer
$\Rightarrow y-x \in$ Integer
$\Rightarrow (y,x) \in R$
$\Rightarrow$ symmetric.

8 (ii)
$(a,a) \in R$ because $a = 1 \times a$, $1 \in$ rational no.

$(x,y) \in R \Rightarrow x = \alpha y$, $\alpha \in$ rational no.
but $y = \frac{1}{\alpha} x$
if $\alpha = 0$ ie $x = 0$ then
$\frac{1}{\alpha}$ does not exist.
$\Rightarrow$ not symmetric
for example $(0,2) \in R$ but $(2,0) \notin R$

If $(a,b) \in R \Rightarrow a = \alpha_1 b$
$(b,c) \in R \Rightarrow b = \alpha_2 c$
$\Rightarrow a = \alpha_1 \alpha_2 c$
$\Rightarrow (a,c) \in R \Rightarrow$ transitive.

33 $(A,A) \in R$ because $A = I^{-1} A I$
where I is identity matrix.

$(A,B) \in R \Rightarrow A = P^{-1} B P$
$\Rightarrow PA = BP$
$\Rightarrow PAP^{-1} = B \Rightarrow (P^{-1})^{-1} A P^{-1} = B$
$\Rightarrow (B,A) \in R \Rightarrow$ Symmetric

if $(A,B) \in R \Rightarrow A = P^{-1} B P$
$(B,C) \in R \Rightarrow B = S^{-1} C S$

$\Rightarrow A = P^{-1} S^{-1} C S P$
$= (SP)^{-1} C SP$
$\Rightarrow (A,C) \in R \Rightarrow$ transitive.

(84) $y = \log_a (x + \sqrt{x^2+1})$

Since $\sqrt{x^2+1} > -x$ so defined for all $x \in$ real no.

$$y = \frac{\ln (x + \sqrt{x^2+1})}{\ln a}$$

$$\frac{dy}{dx} = \frac{1}{\ln a}\left[\frac{1\left(1 + \frac{1 \times 2x}{2\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}\right] = \frac{1}{\ln a}\left(\frac{\sqrt{x^2+1} + x}{x + \sqrt{x^2+1}}\right)\frac{1}{\sqrt{x^2+1}}$$

$$= \frac{1}{\ln a\sqrt{x^2+1}} > 0$$

$\Rightarrow$ y is increasing

as $x \to -\infty \Rightarrow x + \sqrt{x^2+1} \to 0$

so $\ln(x + \sqrt{x^2+1}) \to -\infty$

as $x \to \infty \Rightarrow x + \sqrt{x^2+1} \to \infty$

$\Rightarrow \ln(x + \sqrt{x^2+1}) \to \infty$

so range $(-\infty, \infty)$

$\Rightarrow$ f(x) is invertible

$y = \log_a [x + \sqrt{x^2+1}] \Rightarrow x + \sqrt{x^2+1} = a^y$

$\Rightarrow \sqrt{x^2+1} = a^y - x$

$\Rightarrow x^2 + 1 = a^{2y} + x^2 - 2xa^y$

$\Rightarrow 2xa^y = a^{2y} - 1$

$\Rightarrow x = \frac{1}{2}(a^y - a^{-y})$

$\Rightarrow f^{-1}(y) = \frac{1}{2}(a^y - a^{-y})$

$\Rightarrow f^{-1}(x) = \frac{1}{2}(a^x - a^{-x})$

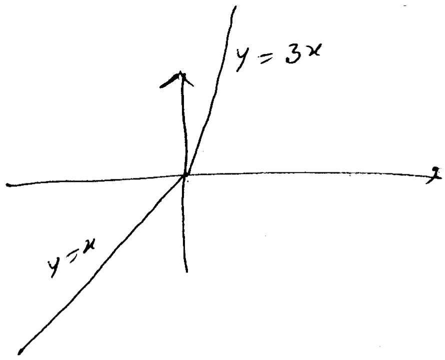
(85) First relation is not symmetric.
second is equivalence

(86) $(x, x) \in R$ because $x \neq x+1 \Rightarrow$ not reflexive

$(x, y) \in R \Rightarrow y = x + 1 \Rightarrow x = y - 1 \Rightarrow (y, x) \in R \Rightarrow$ not symmetric

Relation T is transitive.

(87) $f(x) = \begin{cases} x, & x \le 0 \\ 3x, & x \ge 0 \end{cases}$



From graph onto & one to One.

(88) Relation is reflexive & symmetric but not transitive.

if $(A, B) \in R \Rightarrow$ word A & B have at least one letter common.

$(B, C) \in R \Rightarrow B, C$ have at least one letter common but it is not necessary that A & C also have at least one letter common $\Rightarrow (A, C)$ need not necessarily belongs to relation

(89) Reflexive & transitive.

(90) Since 2 have two images $\Rightarrow$ not function

$(2,3) \in R$ but $(3,2) \notin R \Rightarrow$ not symmetric

$(1,3) \in R$, $(3,1) \in R$ but $(1,1) \notin R \Rightarrow$ not transitive.