[Physics Paper Code : 3]

DISHA CLASSES
JEE (Advance) Paper-2 Solutions

1. (B)(D) $-\frac{2GMm}{L}+\frac{1}{2}mv^2=0$

2. (A)(D) $x = A\sin\omega t$

$v = A\omega\cos\omega t$

$\frac{u_0}{2} = u_0\cos\omega t \Rightarrow \omega t = \frac{\pi}{3}$

$\Rightarrow t = \frac{\pi}{3\omega} = \frac{\pi T}{3\cdot 2\pi} = \frac{T}{6}$

The time to go to the wall & come back $= 2t = \frac{T}{3}$

$= \frac{2\pi}{3}\sqrt{\frac{m}{k}}$ So option B is wrong

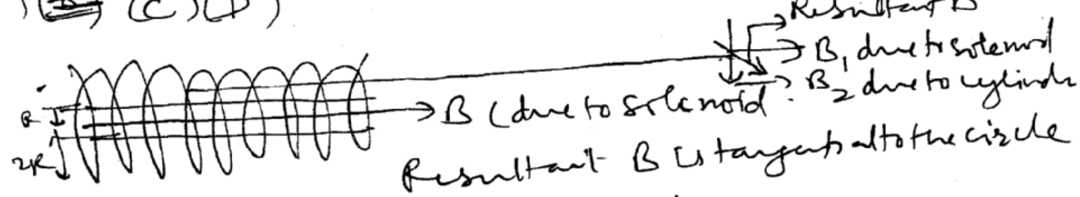
time to reach the other extreme (the max. compression of the spring) is $\frac{T}{4}+\frac{2\pi}{3}\sqrt{\frac{m}{k}}$, $T = 2\pi\sqrt{\frac{m}{k}}$

So option C is wrong

time to get back to the mean position is

$= \frac{T}{2}+\frac{T}{3}$ So D is correct.

3. (A) ~~(B)~~ (C)(D)



Resultant B is tangential to the circle but not along the axis.

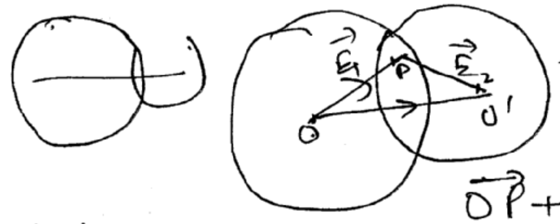
4. (A)(B) (A) $f_2 = \left(\frac{V-\omega+u}{V-\omega-u}\right)f_1$ (B) $f_2 = f_1\left(\frac{V+\omega+u}{V+\omega-u}\right)$

5. (D) $d = \frac{\lambda}{2}\operatorname{cosec}\theta \Rightarrow \Delta d = -\frac{\lambda}{2}\cot\theta\operatorname{cosec}\theta\,\Delta\theta$

though $\Delta\theta$ is constant but $\cot\theta$ & $\operatorname{cosec}\theta$ both decrease as θ increases

$\frac{\Delta d}{d} = \cot\theta\,\Delta\theta$

6 (C) & (D)



for any position of P
$\vec{OP} + \vec{PO'} = \vec{OO'}$ (constant)

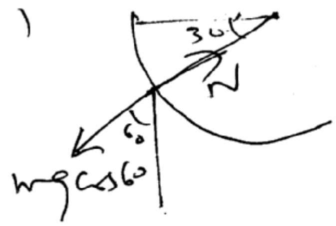
7. (A) (B) (C) (D)

8. (A) & (C) $z = 2$ & $n = 3$

so $\frac{1}{\lambda_1} = 4R\left(\frac{1}{1} - \frac{1}{9}\right)$ & $\frac{1}{\lambda_2} = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$

9. (B) Loss of P.E − Work done against friction
$= \frac{1}{2}mv^2$

10 (A)



$N - mg\cos 60 = \frac{mv^2}{R}$

11. (B) $\cancel{0.2}$ $20 \times 0.4 = 16\ \omega$
$600 \times 10^3 = 4000 \times i \Rightarrow i = 150$

so $100 \times \frac{i^2R}{P_{input}} = 30\%$.

12 (A) input voltage of step down transformer $= 4000 \times 10$
output $= 200$.

so $\frac{N_p}{N_s} = \frac{4000 \times 10}{200} = 200$

13. (B) $\pi R^2 B = E \times 2\pi R$ $\left[\frac{d\phi}{dt} = \int \vec{E} \cdot d\vec{l}\right]$

14 (C) $E \times Q = F$ & $\tau = F \times R$
$\tau \times \Delta t =$ change of angular momentum
so change of Magnetic Moment $= \gamma \frac{BQR^2}{2}$

Q.15 - (C) $m({}_1^2H) + m({}_2^4He) > m({}_3^6Li)$

Q.16(B) ${}_{84}^{210}Po \rightarrow {}_{82}^{206}Pb + {}_2^4He$ Calculate mass defect then Energy released.

Q.17 (D)

Q.18. (C)

Q.15 (A)

Q.20 (C)

Paper – 2 CHEMISTRY

[Chemistry Paper Code : 3]

(21) → B

$$Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4^{2-}$$

s

$0.1+2s \approx 0.1$ $\quad s$

$$1.1\times10^{-12} = (0.1)^2 s$$

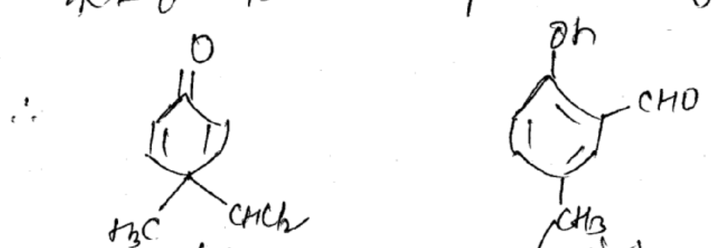
$s = 1.1\times10^{-10}$ answer.

(22) → B, D.

$:CCl_2$ Dichloro (singlet) carbene is E^+.

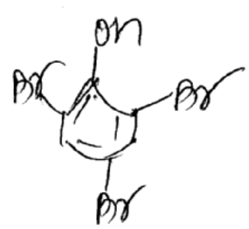
$Ar-O^{\ominus}$ is o & p- directing.

∴



(minor) (major)

(23) → B.



(24) → C.

as trisubstitution occurs in basic medium

mono " occurs in acidic medium.

(2)

(25) → A, C, ~~B~~, D

(26) → A, B

$$_4Be^9 + X \longrightarrow {}_4Be^8 + Y.$$

if $X = {}_0\gamma^0$ then $Y = {}_0n^1$

if $X = {}_1P^1$ then $Y = {}_1D^2$.

(27) → C, D.

(28) → A, B, D

section – 2.

(29) → A

$$Pb^{2+} + 2Cl^{\ominus} \longrightarrow PbCl_2 \text{ (white ppt)}$$

soluble in hot water.

(30) → D.

$$\underset{(P)}{Pb^{2+}} / \underset{(Q)}{Cr^{3+}} \xrightarrow{HCl} \underset{\text{white ppt}}{PbCl_2} + CrCl_2$$

$CrCl_2 \xrightarrow{(H_2S / \text{mineral acid})}$ No effect.

$CrCl_2 \xrightarrow{(H_2S / \text{Ammonical soln})} \underset{\text{(Green ppt)}}{Cr(OH)_3\ (R)}$

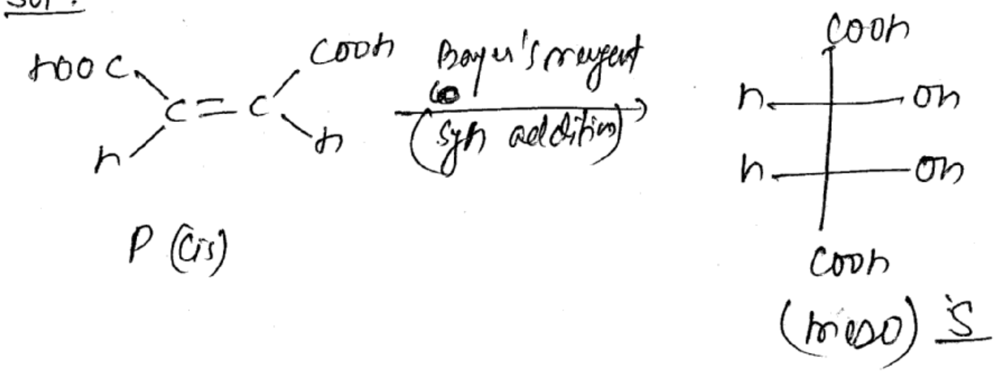
$Cr(OH)_3\ (R) \longrightarrow \underset{(S)}{Na_2CrO_4}$ (yellow soln)

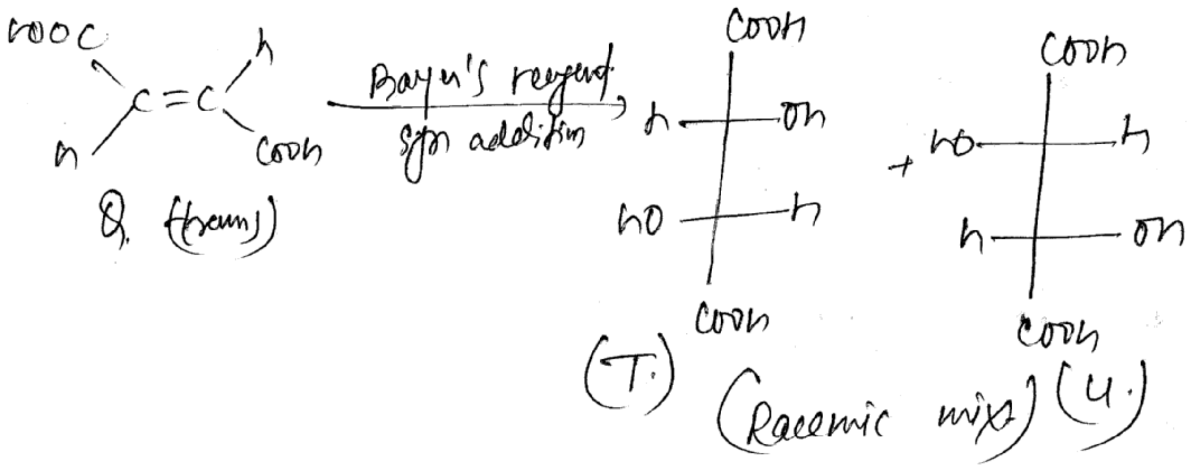
(3.)

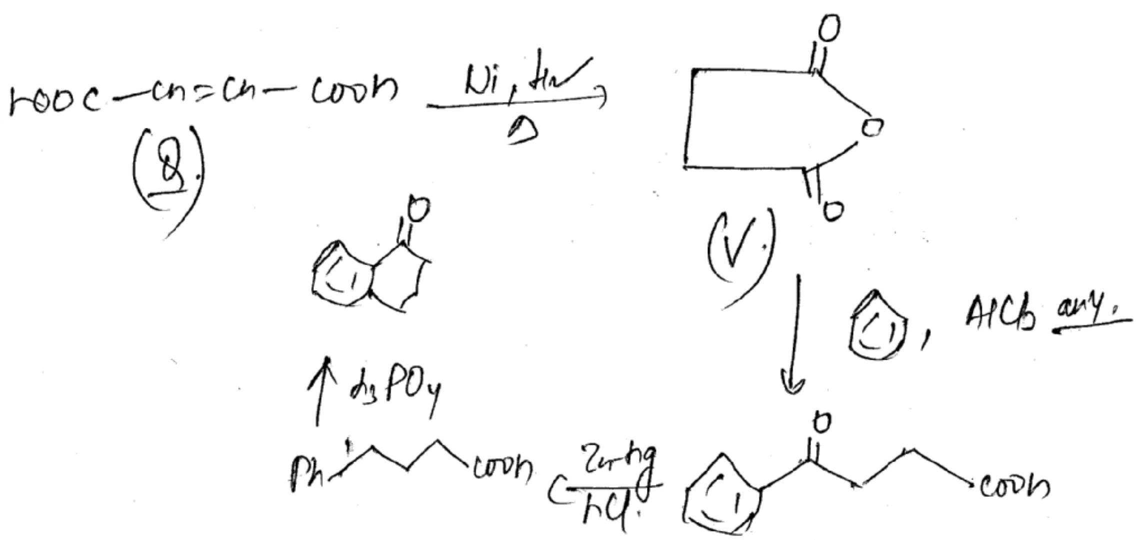
(31) → B

(32) → A.

Solⁿ:







(4.)

33 → C

34 → B.

35 → A.

36 → A.

37 → D

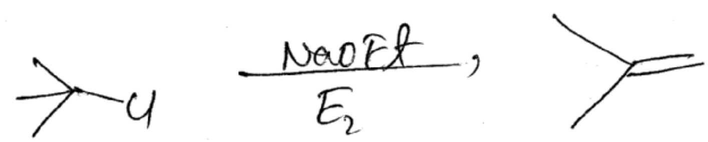
(P) $2PbO_2 + H_2SO_4 \xrightarrow{\Delta} 2PbSO_4 + O_2 + 3H_2O$

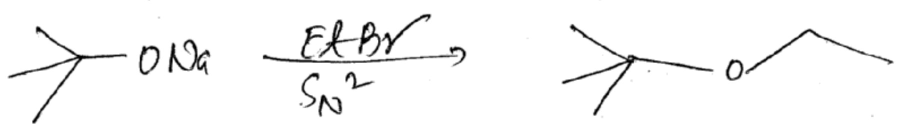
(Q) $Na_2S_2O_3 + H_2O \xrightarrow{Cl_2} NaHSO_4 + HCl$.

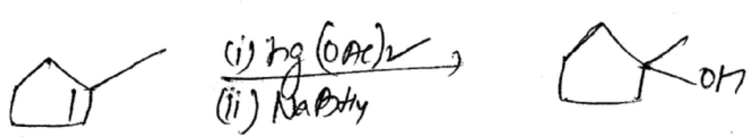
(R) $N_2H_4 \xrightarrow{I_2} NaHSO_4 + HCl$.

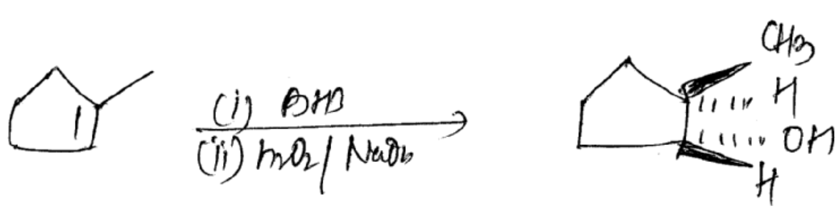
(S) $XeF_2 \xrightarrow{NO} Xe + NOF$.

38 → A.

P 

Q 

R 

S 

(5)

(39) A.
P → 3, Q – 4, R → 2, S → 1.

(P) $Et_3N + CH_3COOH \longrightarrow Et_3\overset{\oplus}{N}H + CH_3COO^{\ominus}$

(Q) $KI\ (0.1M) + AgNO_3\ (0.01M) \longrightarrow AgI\ (s) + KNO_3\ (aq)$

(R) $CH_3COOH + KOH \longrightarrow CH_3COOK + H_2O$

(S) $NaOH + HI \longrightarrow NaI(aq) + H_2O$

(40) → D.
P – 3, Q – 4, R → 1, S → 2.

JEE-ADVANCE
Math Solutions PAPER-1 [Paper Code : 3]

Q41

$$\lim_{n\to\infty} \frac{n^{a+1}\cdot\frac{1}{n}\left[\left(\frac{1}{n}\right)^a + \left(\frac{2}{n}\right)^a + \cdots \left(\frac{n}{n}\right)^a\right]}{(n+a)^{a-1}\left[n^2 a + \frac{n(n+1)}{2}\right]}$$

$$\lim_{n\to\infty} \frac{n^{a+1}}{(n+a)^{a-1}\left[n^2 a + \frac{n(n+1)}{2}\right]} \int_0^1 x^a\,dx$$

$$\lim_{n\to\infty} \frac{1}{\left(1+\frac{a}{n}\right)^{a-1}\left[a + \left(\frac{1+\cancel{\frac{1}{n}}}{2}\right)\right]} \cdot \frac{1}{a+1}$$

$$\frac{1}{1\left[a+\frac{1}{2}\right]}\cdot\frac{1}{a+1} = \frac{1}{60}$$

$$\Rightarrow \frac{(a+1)(2a+1)}{2} = 60 \Rightarrow 2a^2 + 3a - 119 = 0$$

$$2a^2 + 17a - 14a - 119 = 0$$

$$\Rightarrow \cancel{2}a(2a+17) - \cancel{14}7(2a+17) = 0$$

$$(2a+17)(a-\cancel{14}7) = 0$$

$$\Rightarrow a = -17/2 \text{ or } 7$$

Q42 Let Eqn. of circle be

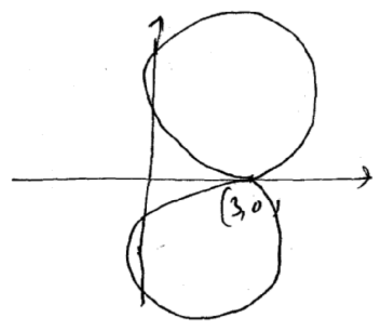
$$(x-3)^2 + y^2 + d y = 0$$

$$x^2 + y^2 - 6x + dy + 9 = 0$$

$$y_{int} = 2\sqrt{\left(\frac{d}{2}\right)^2 - 9} = 2\sqrt{7}$$

$$\Rightarrow \frac{d^2}{4} - 9 = 7$$

$$d^2 = 64 \Rightarrow d = \cancel{\pm 4\sqrt{2}}\ \pm 8$$



Eqn. of circles $\cancel{(x-3)^2}$ $x^2 + y^2 - 6x \pm 8y + 9 = 0$

Q43 $\frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$, $\frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

$$(\vec{a}-\vec{c})\cdot(\vec{b}\times\vec{d}) = 0$$

$$\Rightarrow \begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0 \Rightarrow (5-\alpha)\left[(3-\alpha)(2-\alpha) - 2\right] = 0$$

$\Rightarrow \alpha = 5$ or $(3-\alpha)(2-\alpha) - 2 = 0$

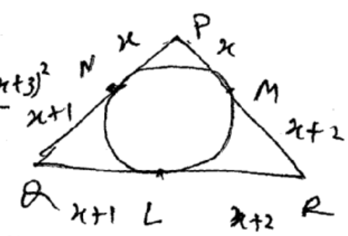
$\Rightarrow \alpha^2 - 5\alpha + 6 - 2 = 0$

$\alpha^2 - 5\alpha + 4 = 0$

$\alpha = 4, 1$

Q44

$\cos P = \frac{1}{3} = \frac{(2x+1)^2 + (2x+2)^2 - (2x+3)^2}{2(2x+1)(2x+2)}$



$\Rightarrow 2(4x^2 + 6x + 2) = 3[4x^2 - 4]$

$\Rightarrow 4(2x^2 + 3x + 1) = 12(x^2 - 1)$

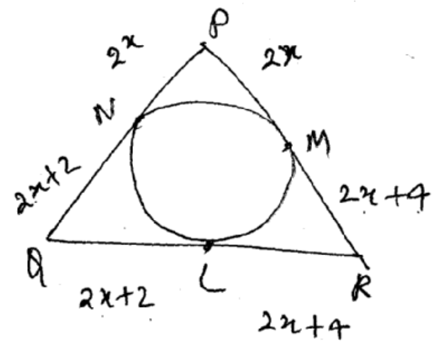
$\Rightarrow 2x^2 + 3x + 1 = 3x^2 - 3$

$\Rightarrow x^2 - 3x - 4 = 0$

x

44

$\cos P = \frac{(4x+2)^2 + (4x+4)^2 - (4x+6)^2}{2(4x+2)(4x+4)}$



$\frac{1}{3} = \frac{16x^2 - 16}{16(2x+1)(x+1)}$

$\Rightarrow \frac{1}{3} = \frac{(x-1)\cancel{(x+1)}}{2x+1} \Rightarrow 2x + 1 = 3x - 3$

$x = 4$

So sides are 18, 20, 22

45

$w = i\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$ $\quad w = \frac{\sqrt{3}}{2} + \frac{i}{2}$

$w^n = i^n\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^n = e^{i\frac{\pi}{2}n} e^{\cdots} = e^{i\pi/6}$

$w^n = e^{in\pi/6} = \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}$

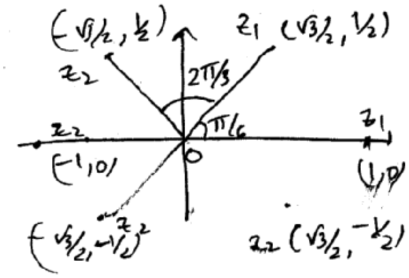
Real part of $P(z w^n)$ is $\cos\frac{n\pi}{6}$, $n = 1, 2, 3, 4$ ——

which can take values $\sqrt{3}/2, 1/2, 0, -1/2, -\sqrt{3}/2, -1$ & 1

for $z_1 \in P \cap H_1$, $Re(z) = \sqrt{3}/2, 1$

$\Rightarrow z_1 = \frac{\sqrt{3}}{2} \pm \frac{i}{2}$ or 1

For $z_2 = -\frac{\sqrt{3}}{2} \pm \frac{i}{2}$ or -1



From figure possible angles

$2\pi/3$, $\frac{5\pi}{6}$

(46) Taking ln

$\ln 3^x = \ln 4^{x-1} \Rightarrow x \ln 3 = (x-1) \ln 4$

$\Rightarrow x = \frac{\ln 4}{\ln 4 - \ln 3}$ — (1)

dividing by $\ln 3 \Rightarrow x = \frac{\frac{\ln 4}{\ln 3}}{\frac{\ln 4}{\ln 3} - 1}$

$= \frac{2 \log_3 2}{2 \log_3 2 - 1}$ (A)

dividing by $\ln 4$

$x = \frac{1}{1 - \frac{\ln 3}{\ln 4}} = \frac{1}{1 - \frac{1}{2}\frac{\ln 3}{\ln 2}} = \frac{1}{1 - \frac{1}{2}\log_2 3}$

$= \frac{2}{2 - \log_2 3}$ (B)

From (B) (C) also follows.

(47) $P = \begin{bmatrix} w^2 & w^3 & w^4 & w^5 & -- & w^{n+1} \\ w^3 & w^4 & w^5 & w^6 & -- & w^{n+2} \\ \vdots & & & & & \\ w^{n+1} & w^{n+2} & & & --- & w^{2n} \end{bmatrix}$

$[P^2_{ij}] = [w^{i+j+2} + w^{i+j+4} + w^{i+j+6} + --- w^{i+j+2n}]$

$= [w^{i+j}(w^2 + w^4 + w^6 + --- w^{2n})]$

If $P^2 \neq 0$

$\Rightarrow w^2 + w^4 + w^6 + --- w^{2n} \neq 0$

$\Rightarrow$ n should not be multiple of 3

so B, C, D.

(48)

$$f(x) = \begin{cases} -2x-(x+2)-|-x-2+2x|, & x\le -2 \\ -2x+x+2-|x+2+2x|, & -2\le x\le 0 \\ 2x+x+2-|x+2-2x|, & x\ge 0 \end{cases}$$

$$= \begin{cases} -3x-2-2+x, & x\le -2 \\ -x+2-|3x+2|, & -2\le x\le 0 \\ 3x+2-|2-x|, & x\ge 0 \end{cases}$$

$$= \begin{cases} -4-2x, & x\le -2 \\ -x+2+3x+2, & -2\le x\le -2/3 \\ -x+2-3x-2, & -2/3\le x\le 0 \\ 3x+2-(2-x), & 0\le x\le 2 \\ 3x+2+2-x, & x\ge 2 \end{cases}$$

$$= \begin{cases} -4-2x, & x\le -2 \\ 2x+4, & -2\le x\le -2/3 \\ -4x, & -2/3\le x\le 0 \\ 4x, & 0\le x\le 2 \\ 2x+4, & x\ge 2 \end{cases}$$

$$f'(x) = \begin{cases} -2, & x<-2 \\ 2, & -2<x<-2/3 \\ -4, & -2/3<x<0 \\ 4, & 0<x<2 \\ 2, & x>2 \end{cases}$$

Ans (A, B)

~~Derivati~~ $f'(x)$ does not exist at $-2, -2/3, 0, 2$
but at $x=2$ sign of $f'(x)$ does not change
so Local max- or minima occurs at $x=-2,-2/3,0$

Q 49 $\quad f''(x) - f'(x) - [f'(x) - f(x)] \geq e^x$

$e^{-x} f''(x) - f'(x) e^{-x} - [f'(x) e^{-x} - f(x) e^{-x}] \geq 1$

$\Rightarrow \frac{d}{dx}[e^{-x} f'(x)] - \frac{d}{dx}[f(x) e^{-x}] \geq 1$

$\Rightarrow \frac{d}{dx}[e^{-x} f'(x) - f(x) e^{-x}] \geq 1 \quad —— (X)$

$\Rightarrow \frac{d}{dx}\frac{d}{dx}[e^{-x} f(x)] \geq 1$

integrating ~~from 0 to x~~

$\Rightarrow \frac{d}{dx} e^{-x} f(x) \geq x + c \quad —— (A)$

again integrating $\Rightarrow e^{-x} f(x) \geq \frac{x^2}{2} + cx + c_1$

$\Rightarrow f(x) \geq e^{x}\left[\frac{x^2}{2} + cx + c_1\right] \quad — (1)$

$f(0) = 0$ from Eq. (1) $\quad f(0) \geq c_1 \Rightarrow 0 \geq c_1$

$\Rightarrow c_1$ can be taken as zero.

$\Rightarrow f(x) \geq e^{x}\left[\frac{x^2}{2} + cx\right] \quad — (2)$

$x = 1 \quad \geq e\left(\frac{1}{2} + c_1\right)$

$\Rightarrow f(1) \geq$

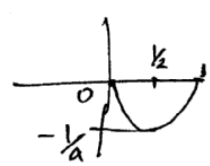
$0 \geq e\left[\frac{1}{2} + c_1\right]$

$\Rightarrow c_1 \leq -\frac{1}{2}$

from (1) $\Rightarrow f(x) \geq e^{x}\left[\frac{x^2}{2} - \frac{1}{2}x\right]$

$\geq \frac{e^x}{2}[x^2 - x]$

$f(x) = x^2 - x$



~~as~~ $0 < x < 1$

$-\frac{1}{4} \leq x^2 - x < 0$

$\Rightarrow -\frac{1}{8}e^{x} \leq \frac{e^x}{2}(x^2 - 2) \leq 0$

so alternative (C) is correct.

(50) ~~from Eq. (A) we can say.~~

$\cancel{\frac{d}{dx} e^{-x} f(x) \geq \frac{1}{2} e^{x} + C}$

from Eq. (1)

$$\frac{d}{dx}\left[e^{-x} f'(x) - e^{-x} f(x)\right] > 0$$

$\Rightarrow$ $e^{-x} f'(x) - e^{-x} f(x)$ is increasing function of x

$h(x) = e^{-x} f(x)$

$h'(x) = e^{-x} f'(x) - e^{-x} f(x)$

$h(x)$ have minima at $x = \frac{1}{4}$

so $h'(\frac{1}{4}) = 0$ & $h'(x)$ is increasing function

so $x \in (0, \frac{1}{4})$

$h'(x) < 0$

$\Rightarrow$ $e^{-x} f'(x) - e^{-x} f(x) < 0$, $x \in (0, \frac{1}{4})$

$\Rightarrow$ $f'(x) < f(x)$, $x \in (0, 1/4)$

so alternative (A)

(51)

Eq. of tangent

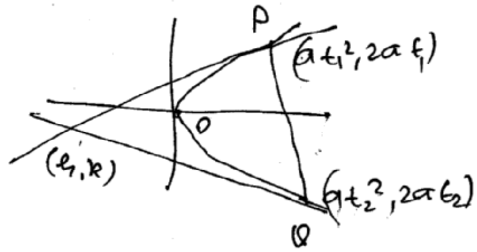
$ty = x + at^2$

$tk = h + at^2$

$at^2 - tk + h = 0$

$t_1 + t_2 = \frac{k}{a}$

$t_1 t_2 = h/a$



For focal chord
$t_1 t_2 = -1$

$\Rightarrow -1 = \frac{h}{a}$

$\Rightarrow h = -a \Rightarrow$

$k = a(t_1 + t_2)$,

(h, k) lies on $y = 2x + a \Rightarrow k = 2h + a$
$= -2a + a = -a$

(52) $\Rightarrow \frac{k}{a} = (t_1 + t_2) \Rightarrow t_1 + t_2 = -1$ & $t_1 t_2 = -1$

$d^2 = a^2(t_1^2 - t_2^2)^2 + 4a^2(t_1 - t_2)^2$

$= a^2(t_1 - t_2)^2 \left[(t_1 + t_2)^2 + 4 \right]$

$= a^2\left[(t_1 + t_2)^2 - 4t_1 t_2 \right]\left[(t_1 + t_2)^2 + 4 \right]$

$= a^2\left[1 + 4 \right]\left[1 + 4 \right] = 25a^2$

$l = 5a$

$m_{OP} = \frac{2}{t_1}$ & $m_{OQ} = \frac{2}{t_2}$

$\tan\theta = \left| \dfrac{\frac{2}{t_1} - \frac{2}{t_2}}{1 + \frac{4}{t_1 t_2}} \right| = \dfrac{|2(t_1 - t_2)|}{|t_1 t_2 + 4|} = \dfrac{2\sqrt{(t_1 + t_2)^2 - 4t_1 t_2}}{|t_1 t_2 + 4|}$

$= \frac{2\sqrt{5}}{3}$

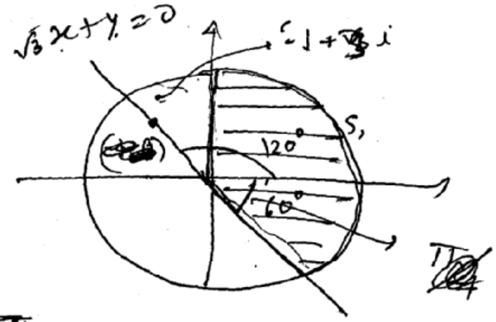
Q 53 $S_1 \rightarrow x^2 + y^2 \le 4^2$

S_2 ~~$\mathrm{Im}\left(\frac{x + iy + 2w}{-2w} \right) > 0$~~

$\mathrm{Im}\left[\left(\frac{x + iy - 1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} \right) \right] > 0$

~~Im~~ $\Rightarrow \frac{(x-1)\sqrt{3} + (y + \sqrt{3})}{4} > 0 \Rightarrow \sqrt{3}x + y > 0$

$S_3 = x > 0$



Shaded portion is S.

$= \frac{\pi r^2}{4} + \frac{\pi r^2}{6}$

$= \cancel{\frac{3\pi r^2}{8}}$

$= \cancel{\frac{3\pi \times 16}{8} = 6\pi}$

$= \frac{\pi \times 16}{4} + \frac{\pi \times 16}{6} = 4\pi + \frac{8\pi}{3} = \frac{20\pi}{3}$

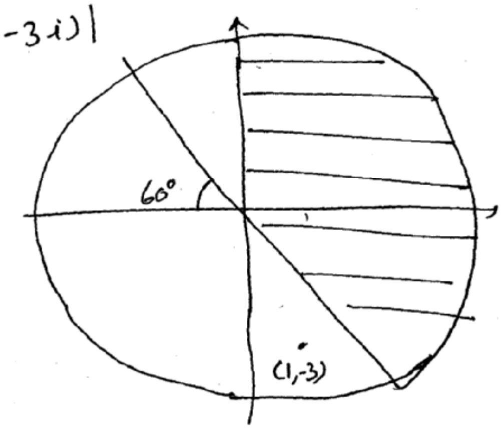
Ans (B)

Ans (B)

54 ~~$|z - (1+3i)| = |z - (1+3i)|$~~

54 $|1 - 3i - z| = |z - 1 + 3i|$

$= |z - (1 - 3i)|$



Minimum value is ⊥ distance from 1, −3 to $\sqrt{3}x + y = 0$

$d = \frac{|\sqrt{3} \times 1 - 3|}{\sqrt{3+1}} = \frac{3 - \sqrt{3}}{2}$

Ans (C)

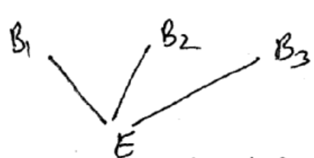
55 $P(WWW) + P(RRR) + P(BBB)$

$\frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$

$= \frac{\cancel{82}\ 41}{108 \times \cancel{6}\ 3} = \frac{41}{324} = \frac{82}{648}$

Ans (A)

56



$P\left(\frac{B_2}{E}\right) = \frac{P(B_2)\, P\left(\frac{E}{B_2}\right)}{\sum P(B_i)\, P\left(\frac{E}{B_i}\right)}$

$$= \frac{\frac{1}{3} \times \frac{2\times 3}{{}^{9}C_2}}{\frac{1}{3}\frac{1\times 3}{{}^{6}C_2} + \frac{1}{3}\times\frac{2\times 3}{{}^{9}C_2} + \frac{3\times 4}{{}^{12}C_2}}$$

$$= \frac{\frac{6}{36}}{\frac{3}{15} + \frac{6}{36} + \frac{12\times 2}{12\times 11}} = \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}}$$

$$= \frac{55}{66+55+60}$$

$$= \frac{55}{181}$$

57

P: $\left[\frac{1}{y^2}\left(\frac{\frac{1}{\sqrt{1+y^2}} + \frac{y\,y}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y} + \frac{y}{\sqrt{1-y^2}}}\right)^2 + y^4\right]^{1/2}$

$$\cancel{= \left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}}\right)^2 + y^4\right]^{1/2} = \frac{1}{y^2}\times y^2(1-y^2)(}$$

$$\left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}}\right)^2 + y^4\right]^{1/2} = \left[\frac{1}{y^2}\times y^2(1+y^2)(1-y^2) + y^4\right]^{1/2}$$

$$= \left[1-y^4+y^4\right]^{1/2} = 1$$

Q: $\cos x + \cos y = -\cos z$ —(1)

$\sin x + \sin y = -\sin z$ —(2)

$(1)^2 + (2)^2 \Rightarrow 1 + 1 + 2(\cos x\cos y + \sin x\sin y) = 1$

$\Rightarrow 2[1+\cos(x-y)] = 1$

$\Rightarrow 4\cos^2\frac{x-y}{2} = 1 \Rightarrow \cos\frac{x-y}{2} = \pm\frac{1}{2}$

R: $\cos 2x\left[\cos\left(\frac{\pi}{4}-x\right) - \cos\left(\frac{\pi}{4}+x\right)\right] = \sec x \sin 2x(\cos x - \sin x)$

$(\cos 2x)\left[2\times\frac{1}{\sqrt{2}}\sin x\right] = \sec x \times 2\sin x\cos x(\cos x - \sin x)$

$\sqrt{2}\sin x\cos 2x = 2\sin x(\cos x - \sin x)$

$2\sin x(\cos x - \sin x) - \sqrt{2}\sin x(\cos^2 x - \sin^2 x) = 0$

$\sqrt{2}\sin x(\cos x - \sin x)[\sqrt{2} - (\cos x + \sin x)] = 0$

$\Rightarrow \sin x = 0$ or $\cos x = \sin x$ or $\sqrt{2} = \cos x + \sin x$

$\Rightarrow \sec x = \pm 1$ or $\tan x = 1$

$\sec x = \pm\sqrt{2}$ or $x = 2n\pi + \pi/4$

$\sec x = \sqrt{2}$

(5) $\cot \cot^{-1} \frac{x}{\sqrt{1-x^2}} = \sin \sin^{-1} \frac{x\sqrt{6}}{\sqrt{1+6x^2}}$

$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{1+6x^2}} \Rightarrow \frac{1}{1-x^2} = \frac{6}{1+6x^2}$

$1+6x^2 = 6-6x^2$

$12x^2 = 5$

$x = \sqrt{\frac{5}{12}} = \frac{1}{2}\sqrt{\frac{5}{3}}$

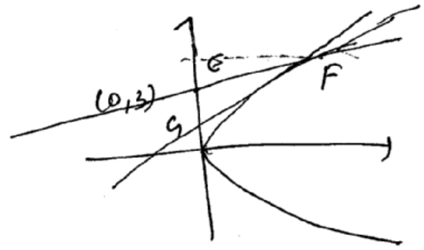
(58) $y^2 = 16x$

let $F(x_0, y_0) = (4t^2, 8t)$

Eqn of FG (Tangent)

$ty = x + 4t^2$

$G = (0, 4t)$



Area of triangle EFG $= \frac{1}{2}\left|(3-4t) \times 4t^2\right|$

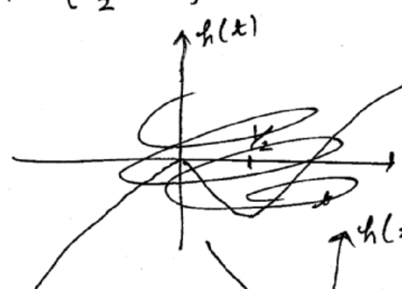
$= 2\left|3t^2 - 4t^3\right|$

Let $h(t) = 3t^2 - 4t^3$

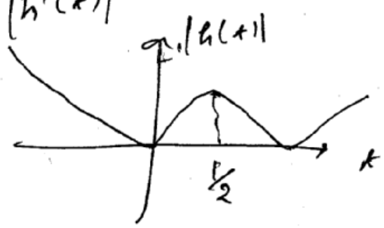
$h'(t) = 6t - 12t^2 = 12t\left(\frac{1}{2} - t\right)$

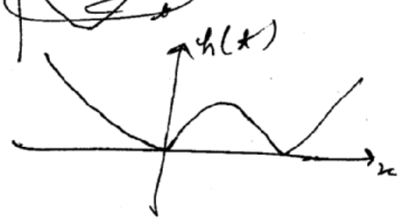
$h'(t)$ — — — + + — — —

t 0 $\frac{1}{2}$



Graph of $|h(t)|$





Clearly area is maximum when t is $\frac{1}{2}$

so $F = (1, 4)$, max. area $= 2\left|\frac{3}{4} - \frac{1}{2}\right| = \frac{1}{2}$

Slope of EF $= \cancel{\frac{2-3}{1-0}}$ $m = \frac{4-3}{1-0} = 1$

(59) $[\bar{a}, \bar{b}, \bar{c}] = 2$

P: $[2(a+b), 3(\bar{b}\times\bar{c}), \bar{c}\times\bar{a}] = 6[a,b,c]^2 = 24$

Q: $[a,b,c] = 5$

$3(\bar{a}+\bar{b})\cdot[(\bar{b}+\bar{c})\times 2(\bar{c}+\bar{a})] =$

$= 6(\bar{a}+\bar{b})\cdot[\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + 0 + \bar{c}\times\bar{a}]$

$= 6\{[\bar{a}, b, c] + 0 + 0 + 0 + 0 + [\bar{b}, \bar{c}, \bar{a}]\}$

$= 12[a,b,c] = 60$

R: $\frac{1}{2}|a\times\bar{b}| = 20$

$\frac{1}{2}|(2\bar{a}+3\bar{b})\times(\bar{a}-b)| = \frac{1}{2}|-2\bar{a}\times b + 3\bar{b}\times a| = \frac{5}{2}|\bar{a}\times b|$

$= 100$

S: $|\bar{a}\times\bar{b}| = 30$

$|(\bar{a}+b)\times a| = |\bar{a}\times b| = 30$

60 For intersection

Point on first line $(2\lambda+1, -\lambda, \lambda-3)$

on second line $(\mu+4, \mu-3, 2\mu-3)$

$2\lambda+1 = \mu+4 \Rightarrow 2\lambda-\mu = 3$

$-\lambda = \mu-3 \Rightarrow \lambda+\mu = 3$

$\Rightarrow \lambda = 2, \mu = 1$

So intersection $= (5, -2, -1)$

Normal to plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= -16\hat{i} + 48\hat{j} + 32\hat{k}$

$= -16(\hat{i} - 3\hat{j} - 2\hat{k})$

Eqn of plane $\bar{r}\cdot(\hat{i}-3\hat{j}-2\hat{k}) = (5\hat{i}-2\hat{j}-\hat{k})\cdot(\hat{i}-3\hat{j}-2\hat{k})$

$x - 3y - 2z = 13$

$x - 3y - 2z = 13$