[Physics Paper Code : 4]

Disha Classes
JEE (Advance) 2013 Physics Solutions
Paper - 1

1 (B) $2A^2 = 2A^2 + 2A^2\cos\phi \Rightarrow \cos\phi = 0 \Rightarrow \phi = \frac{(2n+1)\pi}{2}$

So $\Delta x = \frac{(2n+1)\lambda}{4}$

2. (B) $\Delta P = \frac{P \times t}{c} = \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^{8}} = 1.0 \times 10^{-17}$ kg m/sec

3. (C) In steady state T will be same throughout.

$Y = \frac{T/A}{\Delta l/l}$ So for thin $\Delta l_1 = \frac{Tl}{AY}$

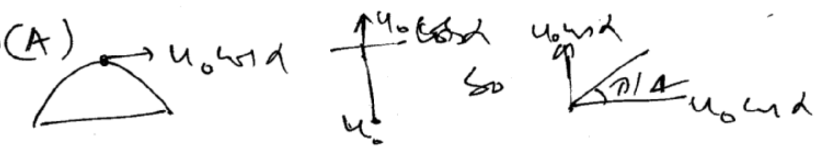
and for thick $\Delta l_2 = \frac{T \times 2l}{4AY}$

$\Rightarrow \frac{\Delta l_1}{\Delta l_2} = 2$

4. (C) $\mu = \frac{\lambda_0}{\lambda} = \frac{3}{2}$ also $m = \frac{v}{u} = -\frac{1}{3} \Rightarrow v = 8$ cm, $u = -24$ cm $\Rightarrow f = 6$ cm

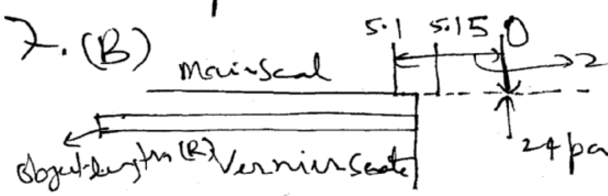
$\frac{1}{6} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{1}{2} \times \frac{1}{R} = \frac{1}{6} \Rightarrow R = 3$ m

5. (A)



6. (D) As the force is along the radius at any pt & instantaneous displacement perp to ~~displacem~~ radius. (tangential)

7. (B)



1 part of the mainscale = 1.05 cm

length upto O = 5.1 + 24 part of the Main Scale
= R + 24 parts of the V scale.

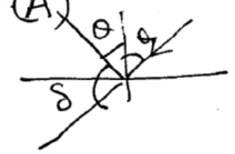
So $6.3 = R + 24 \times \frac{2.45}{50}$

So $R = 5.124$ cm.

8. D $\dfrac{p_1}{p_2} = \dfrac{n_1}{n_2} = \dfrac{4}{3}$ so $\dfrac{m_1}{m_2} = \dfrac{n_1 \times \text{atomic mass}}{n_2 \times \text{atom mass}} = \dfrac{4}{3} \times \dfrac{2}{3}$
$= \dfrac{8}{9}$

9(A) $R_1 = \frac{3}{2}R$, $R_2 = \frac{R}{3}$ so $\dfrac{\Delta\theta}{\frac{3}{2}R} \times 9 = \dfrac{\Delta\theta}{R/3} \times t \Rightarrow t = 2\text{sec}$

10 (A)



Angle between the incident direction & reflected dir^n $= 180 - 2\theta$

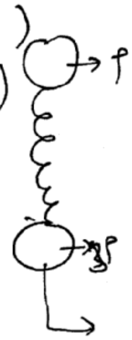
$\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j}) \cdot \frac{1}{2}(\hat{i} - \sqrt{3}\hat{j}) = \frac{1}{4} \left(-\frac{3}{4}\right) = \frac{1}{4} - \frac{3}{4}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}\sqrt{\frac{1}{4} + \frac{3}{4}} \cos\delta$

$\Rightarrow \cos\delta = -\dfrac{2}{4 \times 1} = -\dfrac{1}{2}$

$\Rightarrow \delta = 120 \Rightarrow 180 - 2\theta = 120 \Rightarrow 2\theta = 60$ or $\theta = 30°$

11. (A)
(D)

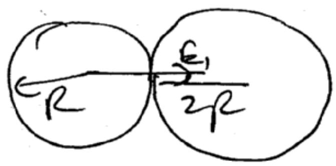


Total wt $= \frac{4}{3}\pi R^3 \times 4\rho g = \frac{4}{3}\pi R^3 \times 2 \times 2\rho g$
= Buoyant force when they are completely merged.

$\frac{4}{3}\pi R^3 \times 3\rho g = \frac{4}{3}\pi R^3 \times 2\rho g + kx$

12. (B) & (D) Initially C_1 will have charge $2CV_0$ which will be equally divided b/w C_1 & C_2 so the upper plate of C_1 will have CV_0 charge, then C_2 will be charged according to the 2nd cell, it's charge will be CV_0 & polarity according to the 2nd cell

13 (D) Both will have the similar charge so the electric field intensity of one will cancel the other.



E_1 (due to smaller)

$$= \frac{K\,\frac{4}{3}\pi R^3 \times \rho_1}{(2R)^2}$$

$$E_2 \text{ (due to the bigger)} = \frac{Kqx}{R^3}$$

$$= \frac{K \times \frac{4}{3}\pi (2R)^3 \times \rho_2 R}{(2R)^3}$$

So $\rho_1/\rho_2 = 4$

14. ~~$y = .01 \sin 2\pi$~~

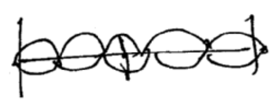
(B),(C) $y = .01 \sin 62.8x \cos 628t$

$$y = 2A \sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T}$$

Comparing $\lambda = .10\ m$ & $f = 100\ Hz$

So $v = f\lambda = .10 \times 100 = 10\ m/sec$

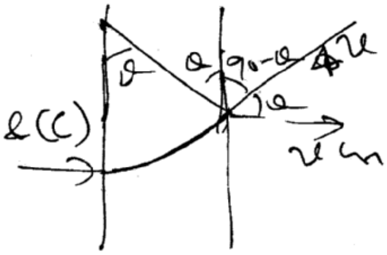
$l = \frac{5}{2}\lambda \Rightarrow l = 0.25\ m$



$2A = .01\ m$

fundamental freq $= \frac{v}{2l} = \frac{10}{2\times 0.25} = 20\ Hz$

Q.15.
(A) & (C)



$v = 4$

$4\sin\theta = 2\sqrt{3}$

$\Rightarrow \theta = 30^\circ$

$T = \dfrac{2\pi M}{QB}$ $\quad t = \dfrac{2\pi M}{QB} \times \dfrac{1}{12} = \dfrac{10}{100}$

$\Rightarrow B = \dfrac{50}{3}\dfrac{\pi M}{Q}$

16. $\dfrac{(N_0 - N)}{N_0} = \dfrac{(N_0 - N_0 e^{-\lambda t})}{N_0} \times 100\% = (1 - e^{-\lambda t}) \times 100$

Ans. (4)

$t = 80 \sec$

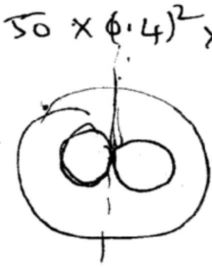
$\& \ \lambda = \dfrac{.693}{T_{1/2}}$

17. (5) $\sqrt{g l_1} = \sqrt{5 g l_2}$

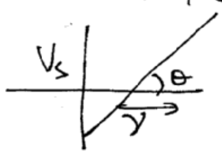
18 (8) $I_1 \omega_1 = I_2 \omega_2$

$\frac{1}{2} \times 50 \times (0.4)^2 \times 10 = \left\{\frac{1}{2} 50 \times (0.4)^2 + 2\left[6.25 \times (0.2)^2 + 6.25 \times (0.2)^2\right]\right\}\omega$

$\Rightarrow \omega = 8$



19. (1) $h\nu = W + eV_s \Rightarrow V_s = \dfrac{h}{e}\nu - \dfrac{W}{e}$



$\tan\theta = \dfrac{h}{e}$ [constant for all metals]

20 (5) $Pt = \frac{1}{2} m v^2$

Paper – I. (1.)

[Chemistry Paper Code : 3]

(21) – C

$$C_6H_{12}O_{6(s)} + 6\,O_2\,(g) \longrightarrow 6\,CO_2\,g + 6\,H_2O\,(l)$$

$$\Delta_c H = 6 \times \Delta H_f\,(CO_2) + 6\,\Delta H_f\,(H_2O) - \left[\Delta H_f\,(C_6H_{12}O_6) + 6\Delta H_f\,(O_2\,g)\right]$$

$$= 6 \times -400 + 6 \times -300 - [-1300 + 0]$$

$$= -4200 + 1300 = -2900\ KJ/mole.$$

1 mole of $C_6H_{12}O_6$ = 180 u.

$\because$ 180 gm of Glucose $\equiv -2900$

$\therefore$ 1 gm " $\equiv \dfrac{-2900}{180}$

$= -16.11\ KJ/gm.$

(22) – (B.)

reacting along S_N2:

$$Ph-\overset{O}{\overset{\|}{C}}-CH_2-Cl > CH_3-Cl > CH_2=CH-CH_2-Cl > CH_3-\overset{Cl}{\overset{|}{CH}}-CH_3$$

(–M Group attachment)

CH_3-Cl ↓ less steric crowding.

$CH_2=CH-CH_2-Cl$ ↓ 1° allyl ↓ crowding less.

(2)

(23) → D.

 has lower Ka value than H_2CO_3.

(24) → (B.)

$P = [\overset{+3}{Fe}F_6]^{3-}$

$Fe^{3+} \Rightarrow 3d^5\,4s^0 \quad n = 5.$

$\mu_{BM} = \sqrt{n(n+2)} = \sqrt{35}.$

$Q = [\overset{2+}{V}(H_2O)_6]^{2+}$ 1

$V^{2+} \Rightarrow 3d^3\,4s^0 \quad n = 3$

$\mu_{BM} = \sqrt{3(3+2)} = \sqrt{15}$

$R = [Fe(H_2O)_6]^{2+}$

$Fe^{2+} \Rightarrow 3d^6\,4s^0 \quad \underline{n = 4.}$

$\mu_{BM} = \sqrt{4(4+2)} = \sqrt{24}.$

(25) → D.

For P, $t_{\frac{1}{2}} = x$ if

$t_{75\%} = 2x.$

means first order rxn. $\therefore \alpha = 1.$

(3)

For Q, $\beta = 0$.

$\therefore\ n = 1 + 0 = 1$.

(26.) → B.

$4HNO_3 \longrightarrow 2H_2O + 2NO_2 + 3O_2$.

(27) → A.

(28) → D.

(29) → B.

In physisorption, enthalpy decreases.

(30) → A.

section – 2

(31.) → B, C, D

As, $\Delta G = -ve$ & $\Delta S_{system} = +ve$ for solⁿ formation

$\Delta S_{surr.} = 0$

$\Delta H = 0$ for ideal solⁿ.

(4)

32 → B, D.

(B) $[Co(NH_3)_4Cl_2]^+$ & $[Pt(NH_3)_2(H_2O)Cl]^+$
both show G.I. individually.

D. $[Pt(NH_3)_3(NO_3)]Cl$ & $[Pt(NH_3)_3Cl]Br$
Ionisation isomerism individually.

33 → A.

ester hydrolysis is first order w.r.t $[H^+]$

$$\frac{R_{HA}}{R_{HX}} = \frac{[H^+]_{HA}}{[H^+]_{HX}}$$

$$\frac{1}{100} = [H^+]_{HA}$$

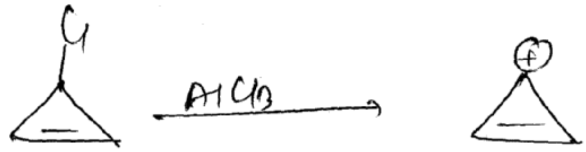
$$HA \rightleftharpoons H^+ + A^{\ominus}$$
$$1-0.01 \quad 0.01 \quad 0.01$$
$$\simeq 1$$

$$K_a = \frac{0.01 \times 0.01}{1} = 10^{-4}$$

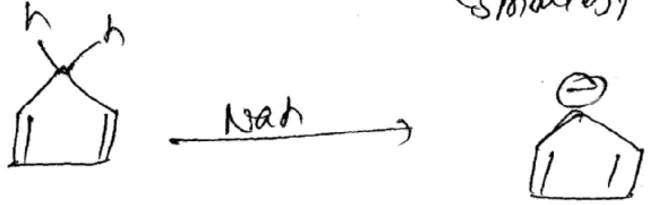
34 → A.

5

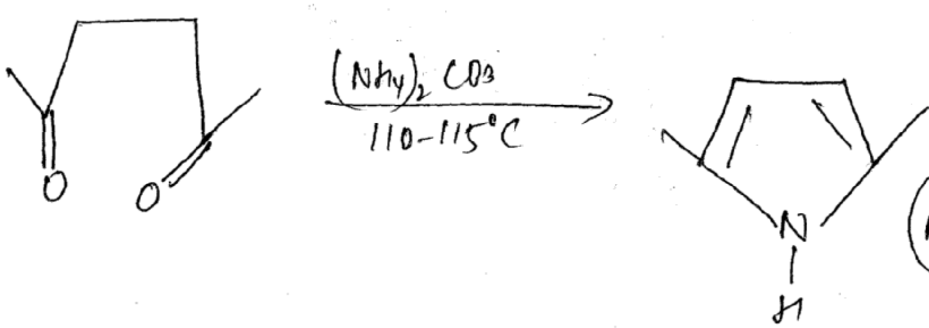
35 → A, B, C, D.



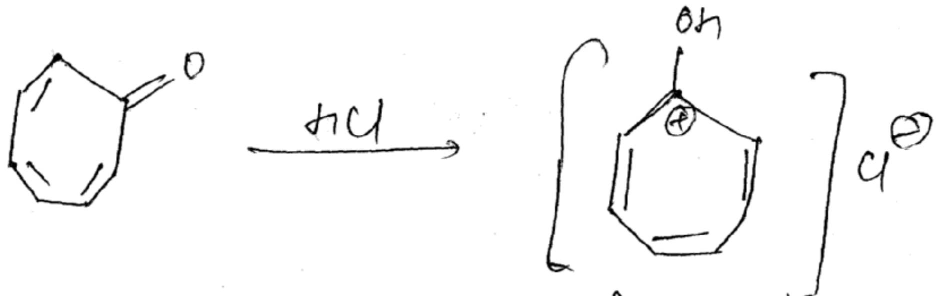
cyclopropyl methyl carbocation.
(Smallest aromatic compd.)



cyclopentadienyl anion.
(Aromatic)



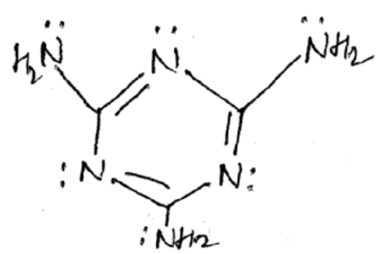
(Aromatic)



(Aromatic)

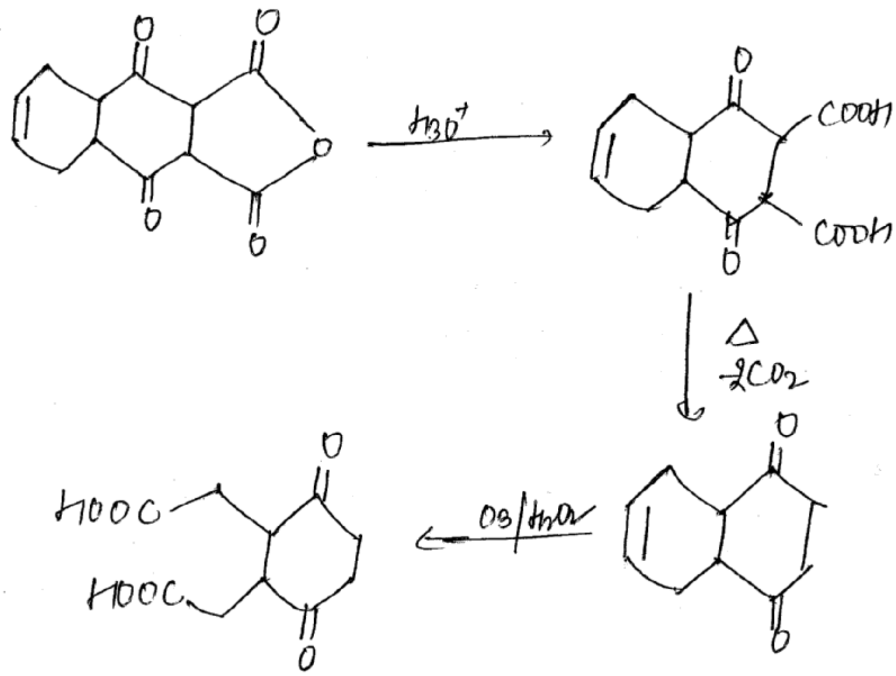
Section-3

36. → 6.



(6)

37. — 2.



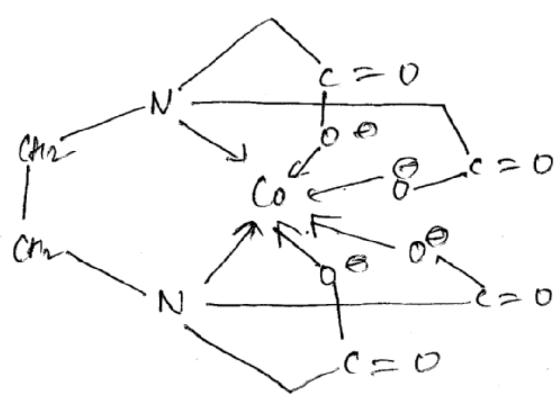
38. → 5.

As we know, $K.E \propto T$

$$\lambda = \frac{h}{\sqrt{2m \times K.E}}$$

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{m_{Ne}\ KE_{Ne}}{m_{He}\ KE_{He}}} = \sqrt{\frac{20 \times 1000}{4 \times 200}} = 5.$$

39. — 8.



7

40 → 4.

Val − Phe − Gly − Ala

Val − Gly − Phe − Ala

Phe − Gly − Val − Ala

Phe − Val − Gly − Ala.

JEE-ADVANCE
Math Solutions Paper 1 [Paper Code : 4]

Q1 $f'(x) - 2f(x) < 0$

$$\int_{1/2}^{x} \frac{d}{dx}\left[e^{-2x} f(x)\right] < 0$$

$$e^{-2x} f(x) - e^{-1} f(1/2) < 0$$

$$e^{-2x} f(x) - e^{-1} < 0$$

$$f(x) < e^{-1} e^{2x}$$

$$\int_{1/2}^{1} f(x)\,dx < e^{-1} \int_{1/2}^{1} e^{2x}\,dx$$

$$< \frac{e^{-1}}{2}\left[e^2 - e\right]$$

$$\int_{1/2}^{1} f(x)\,dx < \frac{1}{2}(e-1)$$

Since $f(x) > 0$

$$\Rightarrow \int_{1/2}^{1} f(x)\,dx > \int_{1/2}^{1} 0\,dx$$

$$> 0$$

So Ans is D.

Q (Q 42) $\frac{dy}{dx} = \frac{y}{x} + \sec \frac{y}{x}$

Let $y/x = t \Rightarrow \frac{dy}{dx} = x\frac{dt}{dx} + t$

$\cancel{t} + x\frac{dt}{dx} = \cancel{t} + \sec t$

$\Rightarrow \cos t\, dt = \frac{dx}{x}$

$\Rightarrow \sin t = \ln x + c$

$\Rightarrow \sin y/x = \ln x + c$

passes through $(1, \pi/6)$

$\Rightarrow \sin \pi/6 = \ln 1 + c \Rightarrow c = 1/2$

so eqn $\sin y/x = \ln x + 1/2$

Ans – A

(Q 43) All lines in alternative passes through $(0,1,2)$ so projection line will pass through $(0,1,2)$

intersection of line & plane: Point on the line $(2t-2, -t-1, 3t)$ lies on

Plane $\Rightarrow 2t-2-t-1+3t = 3$

$4t = 6$

$t = 3/2$

Intersection $(1, -5/2, 9/2)$

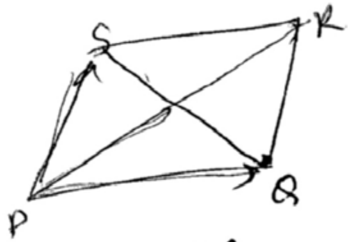
Parallel vector to Projection line $= (1-0)\hat{i} + (-5/2-1)\hat{j} + (9/2-2)\hat{k}$

$= \hat{i} - 7/2\,\hat{j} + 5/2\,\hat{k}$

Eqn of projection line $\frac{x}{1} = \frac{y-1}{-7/2} = \frac{z-2}{5/2}$

or $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

Q44



$$\overline{PQ} = \frac{\vec{PR} + \vec{SQ}}{2} = 2\hat{i} - \hat{j} - 3k$$

$$PS = \frac{\vec{PR} - \vec{SQ}}{2} = \hat{i} + 2\hat{j} + k$$

$$\text{Volume} = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 2(6-2) + 1(3-1) - 3(2-2)$$

$$= 8 + 2 = 10$$

Ans (c)

Q 45

$$\sum_{n=1}^{23} \cot^{-1}\left(1 + 2\frac{n(n+1)}{2}\right)$$

$$= \sum_{n=1}^{23} \tan^{-1}\frac{1}{1+n(n+1)}$$

$$= \sum_{n=1}^{23} \left(\tan^{-1}(n+1) - \tan^{-1} n\right)$$

$$= \tan^{-1} 24 - \tan^{-1} 1$$

$$= \tan^{-1}\frac{23}{1+24\times 1} = \tan^{-1}\frac{23}{25}$$

$$= \cot^{-1}\left(\frac{25}{23}\right)$$

so $\cot\left(\cot^{-1}\frac{25}{23}\right) = \frac{25}{23}$ Ans (B)

Q 46

$$ax + by + c = 0$$

$$bx + ay + c = 0$$

$$(a-b)x + (b-a)y = 0$$

$$\Rightarrow x = y$$

$$ax + bx + c = 0$$

$$x = -\frac{c}{a+b}$$

pt. of intersection $= \left(-\frac{c}{a+b}, -\frac{c}{a+b}\right)$

$$\left(1 + \frac{c}{a+b}\right)^2 + \left(1 + \frac{c}{a+b}\right)^2 < (2\sqrt{2})^2$$

$$\Rightarrow 2(a+b+c)^2 < 8(a+b)^2$$

$$\Rightarrow (a+b+c)^2 - 4(a+b)^2 < 0$$

$$\Rightarrow (a+b+c)^2 - (2a+2b)^2 < 0$$

$$\Rightarrow (3a+3b+c)(-a-b+c) < 0$$

$\Rightarrow$ since $a, b, c > 0 \Rightarrow$

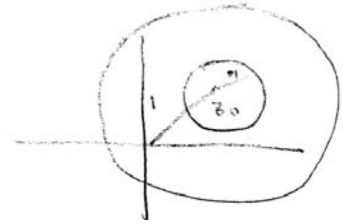
$3a+3b+c > 0 \Rightarrow -a-b+c < 0$ (Ans A)

(Q47) $|\alpha - z_0| = r$

Let $\alpha = a + bi$

$\frac{1}{\alpha} = \frac{1}{a+ib}$

$= \frac{a-ib}{a^2+b^2}$



$(a-x_0)^2 + (b-y_0)^2 = r^2$

$\left(\frac{a}{a^2+b^2} - x_0\right)^2 + \left(\frac{b}{a^2+b^2} + y_0\right)^2 = 4r^2$

$a^2 + b^2 + x_0^2 + y_0^2 - 2ax_0 - 2by_0 = r^2$

$\frac{1}{a^2+b^2} + x_0^2 + y_0^2 - \frac{2ax_0}{a^2+b^2} - \frac{2by_0}{a^2+b^2} = 4r^2$

$a^2+b^2 - 2ax_0 - 2by_0 + 1 + r^2/2 = r^2$

$a^2+b^2 - 2ax_0 - 2by_0 = \frac{r^2}{2} - 1$

$\frac{1}{a^2+b^2} - \frac{(2ax_0+2by_0)}{a^2+b^2} = \frac{7}{2}r^2 - 1$

$\frac{1}{a^2+b^2} - \frac{\frac{r^2}{2} - 1 - (a^2+b^2)}{a^2+b^2} = 7/2\, r^2 - 1$

$-1 + \frac{r^2}{2(a^2+b^2)} = \frac{7}{2}r^2 - 1$

$a^2 + b^2 = \frac{1}{7}$

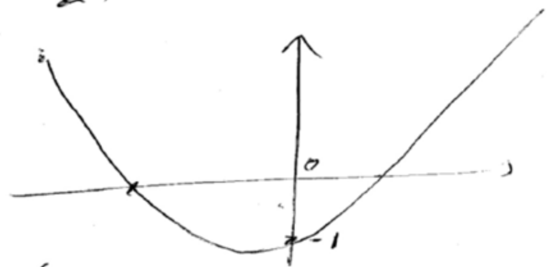
Q48 Let $f(x) = x^2 - x\sin x - \cos x$

$f'(x) = 2x - x\cos x - \sin x + \sin x$

$= 2x - x\cos x = x(2 - \cos x)$

$f'(x)$ − − − − + + + +
x —————— 0 ——————

Graph of $f(x)$



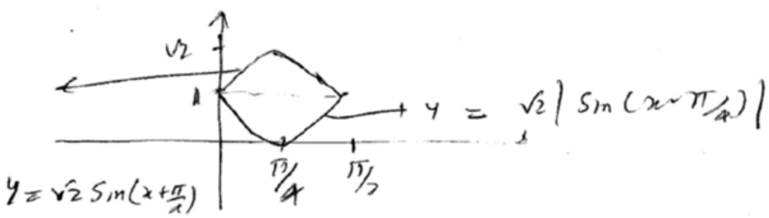
$f(x)$ is decreasing in $(-\infty, 0)$ & ↑ in $(0, \infty)$

graph of $f(x)$ is as shown so $f(x) = 0$ at two values of x

Ans (C)

49) $y = \sqrt{2}\sin(x + \pi/4)$

& $y = \sqrt{2}\,|\sin(x - \pi/4)|$



$$\text{Area} = \cancel{2}\int_0^{\pi/2} \left(\sin x + \cos x - |\cos x - \sin x|\right)dx$$

$$= \cancel{2}\left[\int_0^{\pi/4}\left[\sin x + \cos x - (\cos x - \sin x)\right]dx + \int_{\pi/4}^{\pi/2}\left(\sin x + \cos x - (\sin x - \cos x)\right)dx\right]$$

$$= \cancel{2}\left[\int_0^{\pi/4} 2\sin x\,dx + \int_{\pi/4}^{\pi/2} 2\cos x\,dx\right]$$

(54) $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$

$= -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 - 9^2 - 10^2 - -$

$= 20 + 72$

$\cancel{S_n - S_{n-1} = 4n^2 + 4(n-1)^2 - ...(...)^2}$

$S_n = (3^2 - 1^2) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2) + - - \quad ((4n)^2 - (4n-2)^2)$

$S_n = 2\left[3 + 1 + 4 + 2 + 5 + 7 + 8 + 6 + \text{———} \quad 4n + (4n-2)\right]$

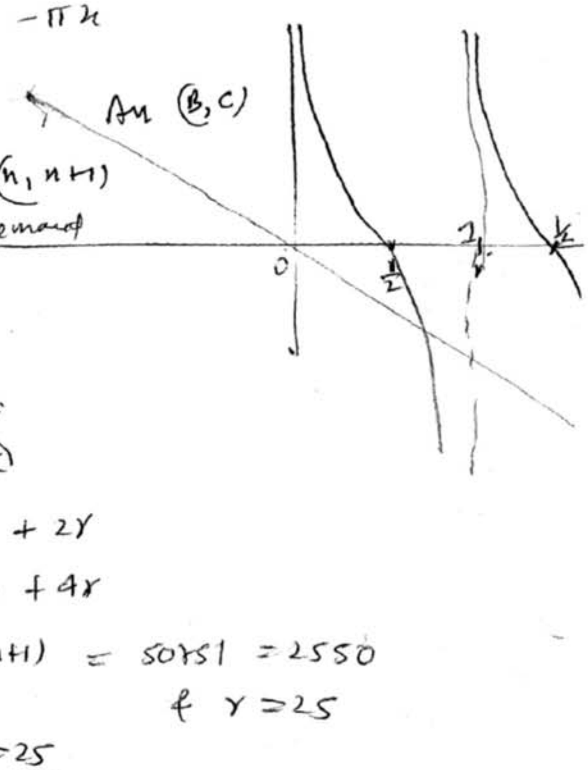
$= 2 \times \frac{4n}{2}\left[2 + (4n-1) \times 1\right] = 4n(4n+1)$

If $S_n = 1056 \Rightarrow n = 8$
no integer n
If $S_n = 1088 \Rightarrow$ no n.
$S_n = 1120 \Rightarrow$ no
$S_n = 1332 \Rightarrow n = 9$

(55) $f'(x) = \cos\pi x + \pi x \sin\pi x = 0$

$\Rightarrow \cos\pi x = -\pi x \sin\pi x$

$\Rightarrow \cot\pi x = -\pi x$



From the figure -
Unique point in $(n + \frac{1}{2}, n+1)$ Ans (B, C)
& hence unique in $(n, n+1)$

(56) Let card no. r & $r+1$ are removed

so $\frac{n(n+1)}{2} - (2r+1) = 1224$

$\cancel{n^2 + n} - 2(2r+1) = 2448$

$\cancel{n^2 + n} - 2(2r+1) - 2448 = 0$

$\frac{n(n+1)}{2} = \cancel{23}\ 1225 + 2r$

$n(n+1) = 2450 + 4r$

If $n = 50$ then $n(n+1) = 50 \times 51 = 2550$
& $r = 25$

so $k = 25$

$2\left[1-\frac{1}{\sqrt{2}}+1-\frac{1}{\sqrt{2}}\right]$

$2\left[2-\sqrt{2}\right]$

$2\sqrt{2}(\sqrt{2}-1)$ Ans B

(50) $P = 1 - P(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c)$

$= 1 - \left(\frac{1}{2}\right) \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$

$= 1 - \frac{21}{256} = \frac{235}{256}$

Ans A

(51) $(N^T M N)^T = N^T M^T N$

so if $M^T = M \Rightarrow N^T M N$ is symmetric

& $M^T = -M \Rightarrow$ " " skew symmetric

so (a) is correct

(B) $(MN - NM)^T = N^T M^T - M^T N^T$

$= NM - MN$

$= -(MN - NB)$ so correct

(C) $(MN)^T = N^T M^T = NM$ so false

(d) $\operatorname{adj}(MN) = (\operatorname{adj} N) \operatorname{adj} M$ so false.

(52) $l_1 = 3\hat{i} - \hat{j} + 4\hat{k} + t(\hat{i} + 2\hat{j} + 2\hat{k})$

$l_2 = 3\hat{i} + 3\hat{j} + 2\hat{k} + s(2\hat{i} + 2\hat{j} + \hat{k})$

l is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= -2\hat{i} + 3\hat{j} - 2\hat{k}$

Eq of l $\frac{x}{-2} = \frac{y}{3} = \frac{z}{-2}$

For intersection of l_1 & l

one l_1 point $(3+t, -1+2t, 4+2t)$

& on l be $(-2\lambda, 3\lambda, -2\lambda)$

$3+t = -2\lambda$ $\Rightarrow$ $\frac{3+t}{-1+2t} = -\frac{2}{3}$

$-1+2t = 3\lambda$

$\Rightarrow$ $2-4t = 9+3t$

$t = -1$

so intersection $(2,-3,2)$

Let point on l_2 be $(3+2s, 3+2s, 2+s)$

$(3+2s-2)^2 + (3+2s+3)^2 + (2+s-2)^2 = 17$

$(2s+1)^2 + (2s+6)^2 + (s)^2 = 17$

$\cancel{9s^2+30s+21=0}$ $9s^2+28s+20=0$

$3s^2+10s+7=0$ $\Rightarrow$ $s = -2, -\frac{10}{9}$

$3s^2+3s+7s+7=0$

$3s(s+1)+7(s+1)=0$

$s = -1, -7/3$ points are $(-1,-1,0)$ or $(\frac{7}{9}, \frac{7}{9}, 8/9)$

~~point $(1,1,1)$ & $(-\frac{5}{3}, -\frac{5}{3}, \frac{1}{3})$~~

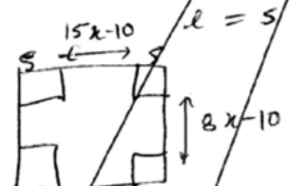
so only correct is (C)

53. let side be $8x$, $15x$

area of 4 removed square $= 100$

$4l^2 = 100$

$l = 5$



Volume $= (15x-10)(8x-10)\times 5$

$= (120x^2 - 230x + 100)\times 5$

$= 50(12x^2 - 23x + 10)$

$\neq 50$

Incorrect.

Q 53. Let sides be $15x$ & $8x$,
length of squares cut be a

$V = (15x-2a)(8x-2a)(a)$

$= 2(15x-2a)(4x-a)(a)$

$= 2[60x^2 - 23xa + 2a^2]a$

$= 2[2a^3 - 23xa^2 + 60ax^2]$

$\frac{dv}{da} = 2[6a^2 - 46xa + 60x^2]$

$\left.\frac{dv}{da}\right|_{a=5} = 0$ given

$\Rightarrow [6 \times 25 - 46x(5) + 60x^2] = 0$

$\Rightarrow 12x^2 - 46x + 30 = 0$

$\Rightarrow 6x^2 - 23x + 15 = 0$

$\Rightarrow 6x^2 - 18x - 5x + 15 = 0$

$6x(x-3) - 5(x-3) = 0$

$x = 5/6$ or 3

if $x = 3$

$\frac{dv}{da} = 2[6a^2 - 138a + 540]$

$\left.\frac{d^2v}{da^2}\right|_{a=5} = 2[12a - 138]\Big|_{a=5}$

$= 2(60-138)$

$= -ve$

so $x = 3$

sides are 45, 24

$x = 5/6$

$\frac{dV}{da} = 2\left[6a^2 - \frac{46 \times 5}{6}a + 60\left(\frac{5}{6}\right)^2\right]$

$\left.\frac{d^2v}{da^2}\right|_{a=5} = 2\left(12a - \frac{115}{3}\right)_{a=5}$

$= 2\left(60 - \frac{115}{3}\right)$

> 0

$\Rightarrow x = 5/6$ volume is not maximum.

Q57 Total no. of three vectors can be selected $= {}^8C_3 = 56$

Vectors are $\hat{i} + \hat{j} + \hat{k}$ – (1)

$-\hat{i} - \hat{j} - \hat{k}$ – (2)

$\hat{i} + \hat{j} - \hat{k}$ – (3) For three vectors to be coplanar

$\hat{i} - \hat{j} + \hat{k}$ – (4) $\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$-\hat{i} + \hat{j} + \hat{k}$ – (5)

$\hat{i} - \hat{j} - \hat{k}$ – (6)

$-\hat{i} - \hat{j} + \hat{k}$ – (7) it is possible only when

$-\hat{i} + \hat{j} - \hat{k}$ – (8) two rows are opposite in sign.

If two vectors are (1) & (2) then third can be any vector from (3) to 8 i.e. no. of ways = 6

then combination of opposite sign are (3) & (7), (4) & (8), (5) & (6) $\Rightarrow$ 3 ways

& ~~thired~~ third can be any of remaining = 6

so Total no. of ways in this case = 6 × 3 = 18

No. of ways ~~where~~ when three vectors are coplanar = 18 + 6 = 24

No. coplanar = 56 − 24 = 32 = 2^5

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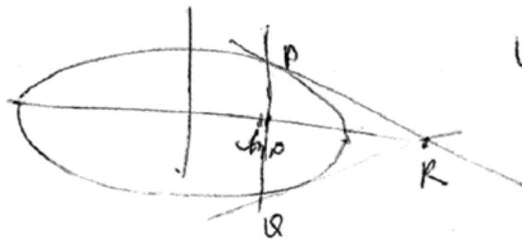

let $h = 2\cos\theta$

~~$R = 3\sin$~~

$P = (2\cos\theta, \sqrt{3}\sin\theta)$

$Q = (2\cos\theta, -\sqrt{3}\sin\theta)$

Eq of Tangent at P

$$\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$$

For R, $y = 0 \Rightarrow x = \frac{2}{\cos\theta}$

so R is $\left(\frac{2}{\cos\theta}, 0\right)$

$$\Delta = \frac{1}{2} \times 2\sqrt{3}\sin\theta\left(\frac{2}{\cos\theta} - 2\cos\theta\right)$$

$$= \sqrt{3}\sin\theta \cdot \frac{2(1-\cos^2\theta)}{\cos\theta} = \frac{2\sqrt{3}\sin^3\theta}{\cos\theta}$$

if $\frac{1}{2} \le h \le 1 \Rightarrow \frac{1}{2} \le 2\cos\theta \le 1 \Rightarrow \frac{1}{4} \le \cos\theta \le \frac{1}{2}$ &

$$\frac{d\Delta}{d\theta} = 2\sqrt{3}\left[\frac{\cos\theta \times 3\sin^2\theta\cos\theta - \sin^3\theta(-\sin\theta)}{\cos^2\theta}\right]$$

$$= 2\sqrt{3}\left[\frac{3\sin^2\theta\cos^2\theta + \sin^4\theta}{\cos^2\theta}\right] > 0$$

$\Rightarrow \Delta$ is increasing w.r.t. to θ & ~~hence w.r.t to h~~

$\Delta_1 = \Delta$ when $\cos\theta = \frac{1}{4} \Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$

$$= \frac{2\sqrt{3}}{1/4} \times \frac{15\sqrt{15}}{4^3} = \frac{15\sqrt{45}}{8} = \frac{45\sqrt{5}}{8}$$

$\Delta_2 = \Delta$ when $\cos\theta = \frac{1}{2} \Rightarrow$

$$\Delta_2 = \frac{2\sqrt{3}}{1/2}\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{4}{8} \times 9 = \frac{9}{2}$$

$$\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = 45 - 8 \times \frac{9}{2} = 9$$

Let terms be ${}^{n+5}C_r$, ${}^{n+5}C_{r+1}$, ${}^{n+5}C_{r+2}$

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$$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{10}$$

$$\frac{(n+5)!\,(r+1)!\,(n+5-r-1)!}{r!\,(n+5-r)!\,(n+5)!} = \frac{1}{2}$$

$$\frac{r+1}{n+5-r} = \frac{1}{2} \Rightarrow 2r+2 = n+5-r$$
$$\Rightarrow n = 3r-3$$

Similarly $\dfrac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \dfrac{10}{14}$

$$\frac{r+2}{n+5-(r+1)} = \frac{5}{7}$$

$$7r+14 = -5r+20+5n$$

$$\cancel{5n-2r+6=0}$$
$$\cancel{5(3r-3)-2r+6=0}$$

$$5n = 12r-6$$
$$5(3r-3) = 12r-6$$
$$3r = 9$$
$$r = 3$$
$$n = 6$$

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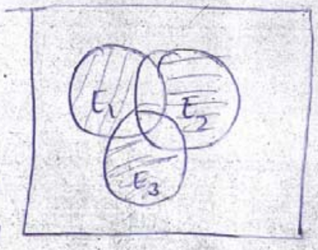
Let $P(E_1) = x$
$P(E_2) = y$
$P(E_3) = z$



$\alpha = x - xy - xz + xyz = x(1-y)(1-z)$

$\beta = y - yx - yz + xyz = y(1-x)(1-z)$

$\gamma = z - zx - zy + xyz = z(1-x)(1-y)$

$p = 1 - [x + y + z - xy - yz - zx + xyz]$

$p = (1-x)(1-y)(1-z)$

$p(\alpha - 2\beta) = \alpha\beta$

$\cancel{(1-x)(1-y)(1-z)}\,[x(1-y)\cancel{(1-z)} - 2y(1-x)\cancel{(1-z)}] = xy\,\cancel{(1-x)(1-y)}(1-z)^2$

$x - xy - 2y + 2xy = xy$

$\Rightarrow \quad x = 2y$

$y = 3z \quad \Rightarrow \quad x = 6z$

$\frac{x}{z} \quad \Rightarrow \quad \frac{x}{z} = 6$