Functions.

①

Q1 $f\{f(x)\} = f\left(\frac{ax}{x+1}\right) = \dfrac{\frac{a\,ax}{x+1}}{\frac{ax}{x+1}+1} = \dfrac{a^2 x}{(a+1)x+1}$

If $f\{f(x)\} = x$ then $a^2 = 1$ & $a+1 = 0$

$\Rightarrow a = -1$

Q2 $y = x + \frac{1}{x} \Rightarrow yx = x^2+1$

$\Rightarrow x^2 - yx + 1 = 0$

$\Rightarrow x = \dfrac{y \pm \sqrt{y^2-4}}{2}$

Since $x \geq 1 \Rightarrow x = \dfrac{y + \sqrt{y^2-4}}{2}$

~~If~~ Since $y \geq 2$

$\dfrac{y - \sqrt{y^2-4}}{2} \leq 1$ so this value of x is rejected

$f^{-1}(y) = \dfrac{y + \sqrt{y^2-4}}{2}$

$f^{-1}(x) = \dfrac{x + \sqrt{x^2-4}}{2}$

Q4 $f(x) = \text{Sgn}(x)$

$g(x) = 1 + x - [x] = 1 + \{x\} \geq 1$

so $f\{g(x)\} = 1$

⑤ ~~So~~ Real solutions of $f(x) = f^{-1}(x)$ is equal to

$f(x) = x$

$\Rightarrow (x+1)^2 - 1 = x$

$\Rightarrow x^2+2x = x \Rightarrow x^2 + x = 0$

$\Rightarrow x = 0, -1$

since $x \geq -1$ so both ~~solution~~ values are solution

⑥ $2^x + 2^y = 2^1 \Rightarrow 2^y = 2 - 2^x$

$\Rightarrow y = \log_2(2-2^x)$

y to be defined $\Rightarrow 2 - 2^x > 0$

$2 > 2^x$

$\Rightarrow 1 > x$

~~$\Rightarrow x \in (1,\infty)$~~

$\Rightarrow x < 1$

$\Rightarrow x \in (-\infty, 1)$

⑦ $B : \quad 1-|x| \geq 0 \Rightarrow |x| \leq 1 \Rightarrow -1 \leq x \leq 1$

$A : \quad |x| - x > 0$

if $x < 0 \Rightarrow -x - x > 0$

$\Rightarrow x < 0$ so $x \in (-\infty, 0)$

if $x \geq 0$ $\quad x - x > 0 \Rightarrow$ not possible

so A is $(-\infty, 0)$

$A \cap B = -1 \leq x < 0 \Rightarrow x \in [-1, 0)$

⑧ $f(x)$ is continuous function

$f'(x) = 2 + \cos x > 0 \Rightarrow f(x)$ is increasing function

$\forall x \Rightarrow f(x)$ is one to one function.

Since $f(x)$ is increasing function so least & greatest values of $f(x)$ is obtained at least & greatest values of x.

as $x \to -\infty \Rightarrow 2x + \sin x \to -\infty$

$x \to \infty \Rightarrow 2x + \sin x \to \infty$

so $f(x) \in (-\infty, \infty)$ = Range

so $f(x)$ is onto.

⑨ $f(x) = \sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)$

$= \sin^2 x + (\sin x \cos \pi/3 + \cos x \sin \pi/3)^2 + \cos x(\cos x \cos \pi/3 - \sin x \sin \pi/3)$

$= \sin^2 x + \left[\frac{1}{4}\sin^2 x + \frac{3}{4}\cos^2 x + 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cancel{\sin x \cos x}\right]$

$+ \cos x\left(\frac{1}{2}\cos x - \cancel{\frac{\sqrt{3}}{2}\sin x}\right)$

$= \frac{5}{4}\sin^2 x + \frac{5}{4}\cos^2 x = \frac{5}{4}$

$g\{f(x)\} = g\{5/4\} = 1$

⑩ $h(-x) = [f(-x) + f(x)][g(-x) - g(x)]$

$= -[f(x) + f(-x)][g(x) - g(-x)]$

$= -h(x) \Rightarrow h(x)$ is odd function

Q. 11

$-\sqrt{3+1} \le \sqrt{3}\sin x + \cos x \le \sqrt{3+1}$ ②

$\Rightarrow -2+4 \le \sqrt{3}\sin x + \cos x + 4 \le 2+4 \Rightarrow$ Range $= [2,6]$

Since function is onto $\Rightarrow$ Range $=$ codomain $\Rightarrow Y = [2,6]$

⑫ (a) onto & one to one (Linear polynomial)

(b) $x^2+2 \geq 2 \Rightarrow$ Range $= [2,\infty) \Rightarrow$ Into

$f(x) = x^2+2 \Rightarrow f'(x) = 2x$ (both +ive & −ive)

$\Rightarrow$ many one.

(c) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}} > 0 \Rightarrow$ one to one

Since $0 \le x < \infty \Rightarrow 0 \le \sqrt{x} < \infty$ so Range is R^+

$\Rightarrow$ onto

~~⑬ $f(x) = \sin\{[x]\}\pi$~~

⑬ $\pi[x]$ is always integral multiple of π for $x \in R$

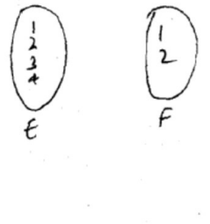
($[x]$ is always integer)

so $\sin(\pi[x]) = 0, \ \forall x \in R$

$\Rightarrow f(x) = 0$ so $f(x)$ is many one & Into

⑭ $f(\pi/2 + x) = f(x)$ so period is $\pi/2$

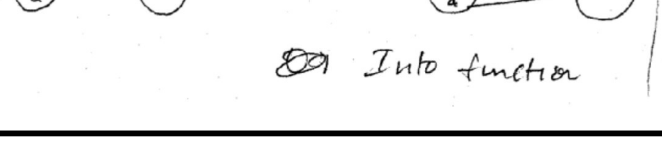
⑮



Each element of E can have image by 2 no. of ways

so Total no of function from E to F
$= 2\times2\times2\times2 = 16$

Out of these 16 function only 2 are Into given below so Total onto function
$= 16-2$
$= 14$



Into function

⑯ $f(x^2)f(y^2) - \frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right) + f(x^2y^2)\right]$

$\frac{2}{2}\cos(\ln x^2)\cos(\ln y^2) - \frac{1}{2}\left[\cos\left(\ln\frac{x^2}{y^2}\right) + \cos\ln(x^2y^2)\right]$

$\frac{1}{2}\left[\cos(\ln x^2 + \ln y^2) + \cos(\ln x^2 - \ln y^2)\right] - \frac{1}{2}\left[\cos\left(\ln\frac{x^2}{y^2}\right) + \cos(\ln x^2y^2)\right]$

$= \frac{1}{2}\left[\cancel{\cos(\ln x^2y^2)} + \cancel{\cos\left(\ln\frac{x^2}{y^2}\right)}\right] - \frac{1}{2}\left[\cancel{\cos\left(\ln\frac{x^2}{y^2}\right)} + \cancel{\cos(\ln x^2y^2)}\right]$

$= 0$

⑰ Period of $\sin nx = \frac{2\pi}{n}$

Period of $\sin\left(\frac{x}{n}\right) = 2\pi n$

so Period of $f(x) = L.C.M.\left(\frac{2\pi}{n}, 2\pi n\right) = 2n\pi$

$2n\pi = 4\pi \Rightarrow n = 2$

⑱ Check from alternative which one satisfies.

⑲ $-1 \le 2x^2 + 3x + 1 \le 1$

$2x^2 + 3x + 2 > 0$, $D = 3^2 - 4 \times 2 \times 2 = -7 < 0$

$\Rightarrow 2x^2 + 3x + 2 > 0, \forall\, x \in R.$ — ①

$2x^2 + 3x + 1 \le 1 \Rightarrow 2x^2 + 3x \le 0$

$2x\left(x + \frac{3}{2}\right) \le 0$

$+ + + \,|\, - - - \,|\, + + +$ ($-\frac{3}{2}$, 0)

$\Rightarrow -\frac{3}{2} \le x \le 0$ — ②

common of ① & ② $\Rightarrow -\frac{3}{2} \le x \le 0$

⑳ $y = g(x)$ is inverse of $f(x)$, find it's inverse.

㉑ $f(x_1) - f(x_2) = \ln\left(\frac{1-x_1}{1+x_1}\right) - \ln\left(\frac{1-x_2}{1+x_2}\right) = \ln\left[\left(\frac{1-x_1}{1+x_1}\right) \times \left(\frac{1+x_2}{1-x_2}\right)\right]$

$= \ln\frac{1 - x_1x_2 - x_1 + x_2}{1 - x_1x_2 + x_1 - x_2} = \ln\left(\frac{1 - \frac{(x_1 - x_2)}{1 - x_1x_2}}{1 + \frac{x_1 - x_2}{1 - x_1x_2}}\right)$

(dividing both Nu & Den. by $1 - x_1x_2$)

$= f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$ so (a) is correct

(b) is also correct ~~because~~ because $\ln\frac{1+x}{1-x} = -\ln\left(\frac{1-x}{1+x}\right)$

(c) $\tan^{-1}\frac{1-x}{1+x} = \tan^{-1}1 - \tan^{-1}x$

$f(x_1) - f(x_2) = \tan^{-1}1 - \tan^{-1}x_1 - \left[\tan^{-1}1 - \tan^{-1}x_2\right]$

$= \tan^{-1}x_2 - \tan^{-1}x_1 = \tan^{-1}\left(\frac{x_2 - x_1}{1 + x_1 x_2}\right)$

$\neq f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$

so (c) & (d) are incorrect.

㉒ $f(x + 1/x) = x^2 + 1/x^2$

$= (x + 1/x)^2 - 2$

$\Rightarrow f(t) = t^2 - 2,$ ~~but~~ where $t = x + 1/x$

since $|x + 1/x| \geq 2$

$\Rightarrow |t| \geq 2$

so $f(t) = t^2 - 2, \quad |t| \geq 2$

$\Rightarrow f(x) = x^2 - 2, \quad |x| \geq 2$

㉓ $0 \leq 2x + 3 \leq 1 \Rightarrow -3 \leq 2x \leq -2$

$-3/2 \leq x \leq -1 \Rightarrow x \in [3/2, -1]$

㉔ (a) let $f(x) = c$, $f(-x) = c \Rightarrow f(x) = f(-x) \Rightarrow$ even function

(b) $f(-x) = -\sin x + \cos x \neq f(x) \Rightarrow$ neither even nor odd.

(c) $f(-x) = \sin\left[\log\left(-x + \sqrt{1+x^2}\right)\right] = \sin\left[\log\left(\frac{\sqrt{1+x^2} - x}{\sqrt{1+x^2} + x}\right)\left(\sqrt{1+x^2} + x\right)\right]$

$= \sin \log\left[\frac{1 + x^2 - \cancel{x^2}}{\sqrt{1+x^2} + x}\right]$

$= \sin \log\left[\frac{1}{\sqrt{1+x^2} + x}\right] = \sin\left[-\log\left(\sqrt{1+x^2} + x\right)\right]$

$= -\sin\left[\log\left(\sqrt{1+x^2} + x\right)\right]$

$= -f(x)$

$\Rightarrow$ odd function

(d) $f(-x) = 1 - x - 2x^3 \neq -f(x) \Rightarrow$ not odd.

(25) $y = 1 + \alpha x$

$\Rightarrow x = \frac{y-1}{\alpha} = f^{-1}(y)$

$\Rightarrow f^{-1}(x) = \frac{x-1}{\alpha} = \frac{x}{\alpha} - \frac{1}{\alpha}$

for $f^{-1}(x)$ to be ~~equal~~ same as that of $f(x)$

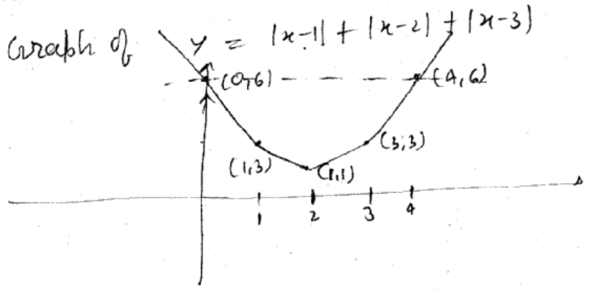
$-\frac{1}{\alpha} = 1$ & $\alpha = \frac{1}{\alpha}$

$\Rightarrow \alpha = -1$ & $\alpha = \pm 1$

$\Rightarrow \alpha = -1$

(26) check from alternatives.

(27) Graph of $y = |x-1| + |x-2| + |x-3|$



from graph

$y \geq 6$

$\Rightarrow x \leq 0$ or $x \geq 4$

so $x \in (-\infty, 0] \cup [4, \infty)$

(28) If $x \in [0, \infty)$ $f(x)$ is continuous (pt. of discontinuity is $x = -1$)

~~let~~ $f'(x) = \frac{1}{(1+x)^2} > 0 \Rightarrow f(x)$ is increasing curve so $f(x)$ is one to one.

$f(0) = 0$, when $x \to \infty$ $\frac{x}{1+x} = \lim_{x \to \infty} \frac{1}{\frac{1}{x}+1} = \frac{1}{0+1} = 1$

so range is $[0, 1)$

$\Rightarrow f(x)$ is one to one & into function.

(29) $g\{f(x)\} = g\{\sin x + \cos x\} = (\sin x + \cos x)^2 - 1$

$= 1 + 2\sin x \cos x - 1 = \sin 2x$

$\sin 2x$ is one to one ~~is~~ when $-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

so only in (b) $f(x)$ is one to one $\Rightarrow f(x)$ is invertible.

(30) $f - g = \begin{cases} -x, & x \in \text{rational no.} \\ x, & x \in \text{irrational no.} \end{cases}$

Image of every rational no is -ive of that $(-x)$

& " " " irrational no. is same number

so $f(x)$ is one to one function
Images ie range contains all rational & irrational no. so
range is $(-\infty,\infty) \Rightarrow f(x)$ is onto

(31) $7-x\,,\ 3-x \in I^{+}$ & $7-x \geq x-3$
$x \leq 5$ — ①

$7-x \geq 0 \Rightarrow x \leq 7$ & $3-x \geq 0 \Rightarrow x \leq 3$ — ②

combining ① & ② $\Rightarrow x = 3, 4, 5$

range $= \{f(3), f(4), f(5)\}$
$= \{{}^{4}P_{0}\,,\ {}^{3}P_{1}\,,\ {}^{2}P_{2}\} = \{1, 3, 2\}$

(32) $g(-x) = \frac{f(-x)}{1+[f(-x)]^2}$, $f(x+y) = f(x)\,f(y)$ — ①
$x = y = 0$
$\Rightarrow f(0) = [f(0)]^2 \Rightarrow f(0) = 1$
sub. in $y = -x$ in ①
$\Rightarrow f(0) = f(x)\,f(-x)$
$1 = f(x)\,f(-x)$
$\Rightarrow f(-x) = \frac{1}{f(x)}$

$= \frac{\frac{1}{f(x)}}{1+\frac{1}{[f(x)]^2}}$

$= \frac{f(x)}{[f(x)]^2+1}$

$= g(x)$

$\Rightarrow g(x)$ is even

(33) $[x+1] \neq 0 \Rightarrow x+1 \notin [0,1)$
$\Rightarrow x \notin [-1,0)$
$\Rightarrow D = R-[-1,0)$

(34) $f(-x) = f(x)$ so $y = f(x)$ is symmetric about y axis $\Rightarrow$
$f(x)$ is many one function.

$f(x) = \frac{e^{x^2}-\frac{1}{e^{x^2}}}{e^{x^2}+\frac{1}{e^{x^2}}} = \frac{e^{2x^2}\overset{+1}{-1}{}^{-1}}{e^{2x^2}+1} = 1-\frac{2}{e^{2x^2}+1}$

$1 \leq e^{2x^2} < \infty \Rightarrow 2 \leq e^{2x^2}+1 < \infty$
$0 < \frac{1}{e^{2x^2}+1} \leq \frac{1}{2} \Rightarrow -1 \leq \frac{-2}{e^{2x^2}+1} < 0$

$\Rightarrow 0 \leq 1-\frac{2}{e^{2x^2}+1} < 1$

$\Rightarrow$ range $\in [0,1)$
$\Rightarrow$ into function

(35) $f(n+1) = 2f(n) + 1$

$f(2) = 2f(1)+1 = 2\times 1+1$

$f(3) = 2[f(2)]+1 = 2(2+1)+1 = 2^2+2+1$

$f(4) = 2(2^2+2+1)+1 = 2^3+2^2+2+1$

$f(5) = 2(2^3+2^2+2+1)+1 = 2^4+2^3+2^2+2+1$

$\Rightarrow f(n) = 2^{n-1}+2^{n-2}+2^{n-3} \text{———} +1$

$= 1+2+2^2 + - - - + 2^{n-1} = \frac{1(2^n-1)}{2-1} = 2^n-1$

(36) $\{x+n\} = \{x\}$ (because $\{x\}$ is periodic of period 1)

so $[x] + \sum_{r=1}^{2000} \frac{\{x\}}{2007} = [x] + \frac{\{x\}}{2007}\times 2007 = x$

(37) $2f(x-1) - f(1/x - 1) = x$ — ①

Replace x by $1/x$

$\Rightarrow 2f(1/x-1) - f(x-1) = 1/x$ — ②

① × 2 + ② ⇒

$3f(x-1) = 2x + 1/x$

$x \to x+1$

$\Rightarrow 3f(x) = 2(x+1) + \frac{1}{x+1}$

$f(x) = \frac{1}{3}\left[2(x+1) + \frac{1}{x+1}\right]$

(38) Replace $2x+3 \to x$

$\Rightarrow f(x) + f(x+4) = 2$ — ①

replacing $x \to x+4$

$f(x+4) + f(x+8) = 2$ — ②

① − ②

$\Rightarrow f(x) - f(x+8) = 0$ so $f(x)$ is periodic of period 8.

$\Rightarrow f(x+8) = f(x)$

(39) $1-x > 0$, $\log_{10}(1-x) \neq 0$, $x+2 \geq 0$

$\Rightarrow \underline{x < 1}$, $1-x \neq 1$ $\Rightarrow \underline{x \neq 0}$, $\underline{x \geq -2}$

combining all $\Rightarrow x \in [-2, 1) - \{0\}$

㊵ $f(x) = \underline{x^2 + 2bx} + 2c^2 + \underline{b^2} - b^2$

$= (x+b)^2 + 2c^2 - b^2 \geq 2c^2 - b^2$

$\Rightarrow f_{min} = 2c^2 - b^2$

Alt. $f_{min} = \frac{-D}{4a} = -\frac{(4b^2 - 4 \times 1 \times 2c^2)}{4}$

$= 2c^2 - b^2$

Similarly $g_{max} = \frac{-D}{4a} = -\frac{(4c^2 - 4 \times (-1)(b^2))}{-4}$

$= c^2 + b^2$

~~$g_{min} \rightarrow f$~~ $f_{min} > g_{max}$.

$\Rightarrow 2c^2 - b^2 > c^2 + b^2$

$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$

㊶ $f\{f(x)\} = f\left(\frac{x}{(1+x^n)^{1/n}}\right) = \dfrac{\frac{x}{(1+x^n)^{1/n}}}{\left(1 + \frac{x^n}{1+x^n}\right)^{1/n}}$

$= \frac{x}{(1+2x^n)^{1/n}}$

Similarly $fofof = f[f\{f(x)\}] = f\left(\frac{x}{(1+2x^n)^{1/n}}\right)$

$= \dfrac{\frac{x}{(1+2x^n)^{1/n}}}{\left(1 + \frac{x^n}{(1+2x^n)^{1/n}}\right)^{1/n}} = \frac{x}{(1+3x^n)^{1/n}}$

Similarly fofofo - - up to n times $= \frac{x}{(1+nx^n)^{1/n}}$

$\int x^{n-2} \cdot \frac{x}{(1+nx^n)^{1/n}} dx = \int \frac{x^{n-1}}{(1+nx^n)^{1/n}} dx$

let $1 + nx^n = t \Rightarrow n^2 x^{n-1} dx = dt$

$= \int \frac{1}{n^2} t^{-1/n} dt = \frac{1}{n^2}\left(\frac{t^{-1/n+1}}{-1/n+1}\right)$

$$= \frac{1}{h^2} \frac{(1+nx^n)^{-1/n+1}}{\frac{-1+n}{n}} + c = \frac{1}{n(n-1)}\left(1+nx^n\right)^{-1/n+1} + c$$

㊷ ~~$f(1) = 0,\ f(2) = -1,\ f(3) = 1,\ f(4) = -2$ - - -~~

~~⇒ every odd intego has it image even natural no.~~

For odd natural no. images are $\frac{n-1}{2}$ which takes values from all +ive integers (including zero)

Images of even natural no. are $(-n/2)$ which are all the -ive integers so range = integers

⇒ f(x) is onto function

f(x) is one to one because odd & even natural numbers do not have samge images.

㊸ $f(x) = \log(\underline{px^3 + px^2} + \underline{qx^2 + qx} + \underline{rx + r})$

$= \log[px^2(x+1) + qx(x+1) + r(x+1))$

$= \log[(x+1)(px^2 + qx + r)]$

For domain $(x+1)(px^2+qx+r) > 0$

Since $q^2 - 4pr = 0 \Rightarrow px^2 + qx + r$ is perfet square

$px^2 + qx + r = p\left(x + \frac{q}{2p}\right)^2$ $\left(\begin{array}{l}\text{roots are } -\frac{b}{2a} \\ = -\frac{q}{2p}\end{array}\right)$

so $p(x+1)\left(x + \frac{q}{2p}\right)^2 \geq 0$

$\Rightarrow x \neq -\frac{q}{2p}$ & $x + 1 > 0$

$x > -1$

so Domain $(-1, \infty) - \{-\frac{q}{2p})$

㊹ ~~$\leq \sqrt{2}(\sin x - \cos x)$~~

$-\sqrt{2+2} \leq \sqrt{2}(\sin x - \cos x) \leq \sqrt{2+2}$

$-2 \leq \sqrt{2}(\sin x - \cos x) \leq 2$

$1 \leq \sqrt{2}(\sin x - \cos x) + 3 \leq 5$

$\log_{\sqrt{5}} 1 \leq \log_{\sqrt{5}}[\sqrt{2}(\sin x - \cos x) + 3] \leq \log_{\sqrt{5}} 5$

$0 \le \log_{\sqrt{3}}\left[\sqrt{2}(\sin x - \cos x) + 3\right] \le 2$

(45) $f(2a-x) = f[a-(x-a)] = f(a)f(x-a) - f(a-a)f(a+x-a)$

$= f(a)f(x-a) - f(0)f(x)$ — ①

In $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$

sub. $x = y = 0$

$\Rightarrow f(0) = [f(0)]^2 - f(a)f(a)$

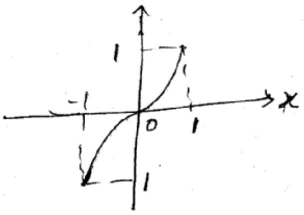
$1 = 1 - [f(a)]^2 \Rightarrow f(a) = 0$

sub. in ①

$\Rightarrow f(2a-x) = 0 - 1f(x) = -f(x)$

(46) (a) $f(x) = \begin{cases} 0, & -1 \le x \le 0 \\ 2x, & 0 \le x \le 1 \end{cases}$ $\Rightarrow$ not a function because $x \in (1/2, 1]$ does not have image in co-domain.

(b) $f(x) = \begin{cases} -x^2, & -1 \le x \le 0 \\ x^2, & 0 \le x \le 1 \end{cases}$ One to one & Onto. $\Rightarrow$ bijective



(c) $f(x) = x+1$ not a function $x \in (0,1]$ does not have image.

(d) $f(x) = \cos\frac{\pi x}{2}$

range $[0,1] \Rightarrow$ into. ~~at~~ as well as many one because $f(-x) = f(x)$

(47) $f(2\pi + x) = \sin(2\pi + x + 3\sin(2\pi + x))$

$= \sin(2\pi + \underline{x + 3\sin x}) = \sin(x + 3\sin x)$

$= f(x)$

$\Rightarrow f(x)$ is periodic of period 2π

$\Rightarrow$ Satement 1 is correct

Statement 2 is false

ex. If $g(x) = x + 3\sin x$ is not periodic but $\sin(x + 3\sin x)$ is period

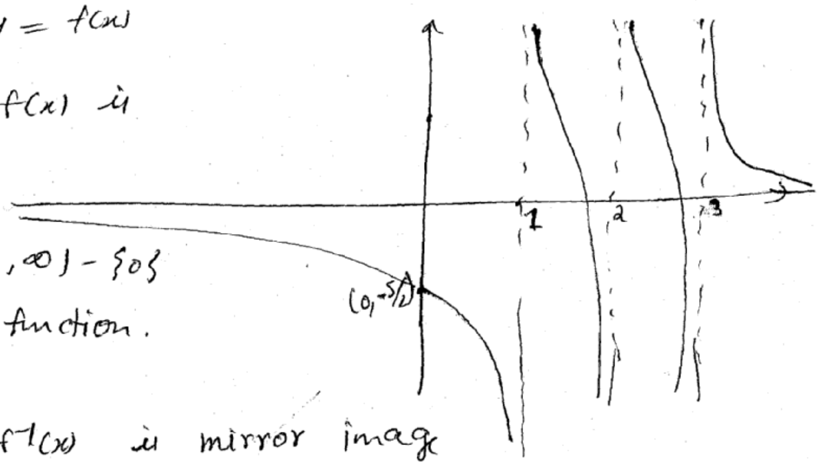
(48) $0 < \sin^2 x \leq 1 \Rightarrow -\infty < \ln \sin^2 x \leq 0 \Rightarrow$ Range $(-\infty, 0]$

so statement ① is correct ② is also correct & correct reason to.

(49) $f'(x) = -\dfrac{1}{(x-1)^2} - \dfrac{1}{(x-2)^2} - \dfrac{3}{(x-3)^2} < 0$

so $f(x)$ is decreasing function of x in the intervals of continuity. ~~But~~ $f(x)$ is discontinuous at $x = 1, 2, 3$

Graph of $y = f(x)$



From graph $f(x)$ is many one

Range $= (-\infty, \infty) - \{0\}$

$\Rightarrow$ into function.

(50) (a) Graph of $f^{-1}(x)$ is mirror image of $y = f(x)$ about line $y = x$

so if $f(x)$ is increasing then $f^{-1}(x)$ is also increasing $\Rightarrow$ (a) is correct.

(b) If $f(x)$ is constant then $f(x)$ is not bijective hence inverse does not exist.

(c) If $f(x)$ & $f^{-1}(x)$ are same then they becomes coinciding so they intersect at all points so false.

(d)

⑦

$f(x) > 0$, $f(x) \cancel{\geq} 1$

$\Rightarrow x \in (-\infty, 1) \cup (2,3) \cup (5,\infty)$ $\Rightarrow x \in (-\infty,-1) \cup (2,3)$

~~(a) If $-1<x<1$ $\Rightarrow f(x)>0$ or ~~$f(x)>1$~~ ,~~

~~so (r)~~

~~(b) $1<x<2$ $\Rightarrow f(x)<0$ or $f(x)>1$ so (q) is correct~~

$f(x) < 0$ $f(x) < 1$

$x \in (1,2) \cup (3,5)$ $x \in (-1,2) \cup (3,\infty)$

If (a) $-1<x<1$ $\Rightarrow f(x) > 0$ & $f(x) < 1$

so (p), (r), (s) matches.

(b) $1<x<2$ $\Rightarrow f(x) < 0$ & $f(x) < 0$

so (q) (s) matches.

(c) $3<x<5$ $\Rightarrow f(x)<0$ $\Rightarrow$ (q), (s) matches

(d) $x>5$ $\Rightarrow f(x)>0$ & $f(x)<1$

so (p), (r), (s) matches.

Comprehension

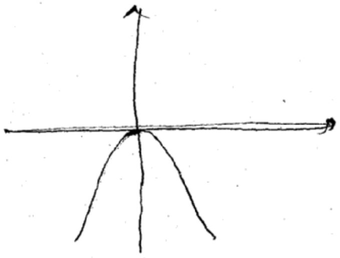
Q 52

(a) If we shift the origin at (1,1) then graph becomes

& curve becomes

$Y+1 = f(X+1)$

$Y = f(X+1) - 1$



If we take mirror ~~image~~ image of this curve about X axis then this is figure (2)

For mirror image about X axis

$-Y = f(X+1) - 1$

$Y = -f(X+1) + 1$

$y = -f(x+1) + 1$

(d) $f(x) = \begin{cases} \dfrac{x}{1-x}, & x \le 0 \\ \dfrac{x}{1+x}, & x \ge 0 \end{cases}$

let $y = \dfrac{x}{1-x}$ if $x \le 0$

$\Rightarrow y \le 0$

$\Rightarrow y - yx = x$

$x = \dfrac{y}{1+y}$, $y \le 0$

let $y = \dfrac{x}{1+x}$

$y + yx = x \Rightarrow x = \dfrac{y}{1-y}$, $y \ge 0$

so $f^{-1}(y) = \begin{cases} \dfrac{y}{1+y}, & y \le 0 \\ \dfrac{y}{1-y}, & y \ge 0 \end{cases} \Rightarrow f^{-1}(y) = \dfrac{y}{1-|y|}$

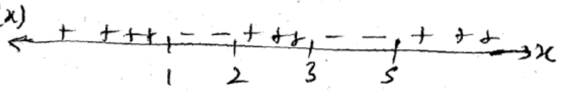
$f^{-1}(x) = \dfrac{x}{1-|x|}$

(51)

$f(x) \ge 0 \Rightarrow \dfrac{x^2-6x+5}{x^2-5x+6} > 0$

$\Rightarrow$ ~~$(x-3)(x-2)$~~ $\dfrac{(x-5)(x-1)}{(x-2)(x-3)} > 0$

Sign of $f(x)$

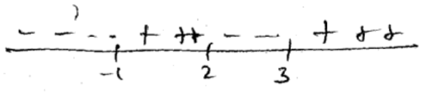


$f(x) > 1$

$\Rightarrow \dfrac{x^2-6x+5}{x^2-5x+6} - 1 > 0$

$\Rightarrow \dfrac{x^2-6x+5-(x^2-5x+6)}{x^2-5x+6} > 0 \Rightarrow \dfrac{-x-1}{(x-2)(x-3)} > 0$

$\Rightarrow \dfrac{x+1}{(x-2)(x-3)} < 0$



$\Rightarrow x \in (-\infty, -1) \cup (2, 3)$

52 (2) If $0 \le x \le 2$ $\Rightarrow 1 \le x+1 \le 3$

Since f(x) is defined for $0 \le x \le 2$

so f(x+1) is also defined for $0 \le x \le 2$

Since $0 \le f(x) \le 1$ $\Rightarrow 0 \le f(x+1) \le 1$

$-1 \le -f(x+1) \le 0$

$0 \le 1 - f(x+1) \le 1$

So required function is $1 - f(x+1)$

52 (3) Shift the origin of figure ① at (2, 0)

$Y + 0 = f(X+2)$

$\Rightarrow Y = f(X+2)$

$y = f(x+2)$

~~53 (4) For domain $0 \le x+2 \le 1$ $\Rightarrow -2 \le x \le 1$~~

52 (4) (a) $0 \le x \le 2$ $\Rightarrow -2 \le -x \le 0$ $\Rightarrow$ Domain of f(-x) is [-2, 0]

range of f(x) is [0, 1] $\Rightarrow$ f(-x) range is [0, 1]

$\Rightarrow$ (a) is incorrect.

(b) $0 \le f(x) \le 1$ $\Rightarrow -1 \le f(x) - 1 \le 0$ so range is [-1, 0]

$\Rightarrow$ (b) is incorrect.

(c) $2 \le f(x) + 2 \le 3$ $\Rightarrow$ (c) is incorrect.

(d) $0 \le x+1 \le 2$ $\Rightarrow -1 \le x \le 1$ $\Rightarrow$ domain [-1, 1]

$0 \le f(x+1) \le 1$ $\Rightarrow -1 \le -f(x+1) \le 0$

$0 \le 1 - f(x+1) \le 1$

$\Rightarrow$ range [0, 1] so (d) is correct.

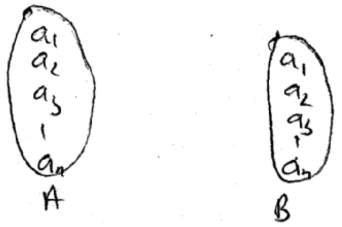
(53) $0 \le \frac{\pi^2}{16} - x^2 \le \frac{\pi^2}{16}$

$0 \le \sqrt{\pi^2/16 - x^2} \le \pi/4$

$0 \le \sin\sqrt{\frac{\pi^2}{16} - x^2} \le \frac{1}{\sqrt{2}}$

$0 \le 3\sin\sqrt{\frac{\pi^2}{16} - x^2} \le 3/\sqrt{2}$

Q54



Each & every element of A can have it's image by n no. of ways

so Total no. of ways in which each & every element of domain have it's image $= n \times n \times n$ - - upto n times

$= n^n$ = Total no. of function

For onto every element of B must have at least one ~~pre in~~ preimage in A ~~so~~ . A & B both contain n elements so every element of A should have different ~~image~~ images so

Total no. of functions $= n(n-1)(n-2) - - 3 \cdot 2 \cdot 1$

$= n!$

Q55 $x^2-8x+18 \neq 0$

because $D = (-8)^2 - 4\times 18 < 0$ so f(x) is continuous function.

$$f'(x) = \frac{(x^2-8x+18)(2x+4) - (x^2+4x+30)(2x-8)}{(x^2-8x+18)^2}$$

$$= \frac{-12x^2+4x+312}{(x^2-8x+18)^2} = \frac{-4(3x^2-x-78)}{(x^2-8x+18)^2}$$

~~$= -4(3x^2 - 13x + 12x$~~

For $3x^2-x-78$

$D = (-1)^2 - 4\times 3(-78) > 0$

so $3x^2-x-78$ is both +ve & -ve

so f'(x) increases as well decreases in its domain

so f(x) is many one.