TYRP-18

(61) $A(\text{Adj}A) = |A|I$ so $d = |A|$

$|A| = \cos^2 x + \sin^2 x = 1$

(62) $|A| = 1 + \tan^2\theta = \sec^2\theta$

$\text{adj}\,A = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

Now $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \cdot \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1+\tan^2\theta & 0 \\ 0 & 1+\tan^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(63) $a_{12} = a_{21} \Rightarrow 2x+3 = x-1$

$\Rightarrow x = -4$

(64) $\det(A_r) = r^2 - (r-1)^2 = 2r-1$

$\sum_{r=1}^{n} \det(A_r) = \sum_{r=1}^{n}(2r-1) = 2\times\frac{n(n+1)}{2} - n = n^2$

(65) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$

$A^2 + I = 0$

$\Rightarrow a^2+bc+1 = 0 \quad -(1) \qquad bc+d^2+1 = 0 \quad -(4)$

$b(a+d) = 0 \quad -(2)$

$c(a+d) = 0 \quad -(3)$

From (2) & (3) $\Rightarrow$ either $b=0, c=0$ or $a+d=0$

if $b=c=0 \Rightarrow a^2+1=0$ & $d^2+1=1$

$\Rightarrow a = \pm i, \; d = \pm i$

$\Rightarrow$ (b) is correct.

If $a+d=0$ then Eqn. can not be solved uniquely.

Alt: check from alternatives

(66) Check from alternatives

$A^2 = B^2 = C^2 = I$

(67) $[p, q, r] = [3\ 0\ 1]\begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix}^{-1}$

Calculate inverse & hence find p, q, r & given value.

(68) Since $a_{11} = a_{22}$ so

$S = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{11} \end{bmatrix}$ is non singular when

$a_{11}^2 - a_{12}a_{21} \neq 0$

Total no of matrices in $S = \underset{\substack{\downarrow \\ \text{no of ways of} \to a_{11}}}{3} \times \underset{\substack{\downarrow \\ a_{12}}}{3} \times \underset{\substack{\downarrow \\ a_{21}}}{3} = 27$

When Matrix is singular then $a_{11}^2 = a_{12}a_{21}$ is possible when (i) $a_{11} = 0$ & at least one of a_{12} & a_{21} is zero

a_{11}	a_{12}	a_{21}
0	0	0
0	1 or 2	0
0	0	1 or 2

$\Rightarrow$ 5 cases.

(ii) when $a_{11} = 1 \Rightarrow a_{21}$ & $a_{12} = 1$

$\Rightarrow$ one case.

(iii) when $a_{11} = 2 \Rightarrow a_{21} = a_{12} = 2$

$\Rightarrow$ one case

so Total ways $= 7$ when S is singular

So no. of nonsingular Matrices in $S = 27 - 7 = 20$

(69) Check the alternatives.

(70) $AB = I$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

both are not equal for any α

(71) $(S_k)^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix}$

$S_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $S_2^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2+2 \\ 0 & 1 \end{bmatrix}$

$(S_2)^3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$

in general $(S_2)^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

$(S_2)^n (S_k)^{-1} = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2n-k \\ 0 & 1 \end{bmatrix}$

$= S_{2n-k}$

(80) (74) $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cos\theta + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\sin\theta$

$= I\cos\theta + J\sin\theta$

(75) $A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 21 & 17 \\ 0 & 37 & 28 \\ 0 & 21 & 16 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 21 & 17 \\ 0 & 37 & 28 \\ 0 & 21 & 16 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 158 & 121 \\ 0 & 269 & 204 \\ 0 & 153 & 116 \end{bmatrix}$

$A^3 - 8A^2 + \alpha A + \beta I = \begin{bmatrix} -7+\alpha+\beta & -10+2\alpha & -15+3\alpha \\ 0 & -27+5\alpha+\beta & -20+4\alpha \\ 0 & -15+3\alpha & -12+2\alpha+\beta \end{bmatrix} = 0$

$\Rightarrow \alpha = 5, \beta = 2$

(76) let $2A + B = X$
$(A - 2B = Y)$

On solving for A & B $\quad 5A = 2X + Y \Rightarrow A = \dfrac{2X+Y}{5}$

$5B = X - 2Y \Rightarrow B = \dfrac{X-2Y}{5}$

$A - B = \dfrac{X+3Y}{5}$

$tr(A) - tr(B) = tr(A-B) = tr\left(\dfrac{X+3Y}{5}\right)$

$= tr\left(\dfrac{X}{5}\right) + tr\left(\dfrac{3Y}{5}\right)$

$= \frac{1}{5}[1+4+2] + \frac{1}{5}[6+9+3]$

$= 5$

N.Roy.

(77) $\det[(\text{adj}A^T)]^T + \det[(\text{adj}A^{-1})^{-1}]$

$\det(\text{adj}A^T) + \dfrac{1}{\det(\text{adj}A^{-1})}$

$= |A^T|^2 + \dfrac{1}{|A^{-1}|^2}$

$= |A|^2 + \dfrac{1}{\frac{1}{|A|^2}} = 2|A|^2 = 2\times 9 = 18$

(78) $|\text{adj}A| = 4(4k+1) - 1(k-2) = 15k+6$

$|\text{adj}A| = |A|^2 = 6^2 = 36$

$\Rightarrow 15k+6 = 36 \Rightarrow k=2$

(79) Differentiating row wise.

$$\frac{dy}{dx} = \begin{vmatrix} \cos x & -\sin x & \cos x - \sin x \\ 23 & 17 & 13 \\ 1 & 1 & 0 \end{vmatrix} + 0 + 0$$

all elements of second row are zero.

$$\frac{d^2y}{dx^2} = \begin{vmatrix} -\sin x & -\cos x & -\sin x - \cos x \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix} + 0 + 0$$

$$\frac{d^2y}{dx^2} + y = \begin{vmatrix} 0 & 0 & 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix} = 23 - 17 = 6$$

(80) Taking $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ common from R_1, R_2, R_3 respectively

$$= \frac{1}{abc}\begin{vmatrix} 1 & abc & a^4 \\ 1 & abc & b^4 \\ 1 & abc & c^4 \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} 1 & 1 & a^4 \\ 1 & 1 & b^4 \\ 1 & 1 & c^4 \end{vmatrix} = 0$$

Now find U_1 & U_2

(81) For non trivial sol. $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$= 1(\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1) = 0$

$\Rightarrow \lambda^3 - \lambda = 0 \Rightarrow \lambda = 0, \pm 1 \Rightarrow$ exactly three values of λ.

(82) $A \text{ adj } A = |A| I = (10a+3b)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$

$AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$

Since given is $A(\text{adj} A) = AA^T$

$\Rightarrow 25a^2 + b^2 = 10a + 3b$ — (1)

$15a - 2b = 0$ — (2)

$10a + 3b = 13$ — (3)

From (2) & (3) $\Rightarrow 10a + 3\left(\frac{15}{2}\right)a = 13$

$\Rightarrow \frac{65a}{2} = 13 \Rightarrow a = 2/5$

& $2b = 15 \times 2/5 \Rightarrow b = 1$

$a = 2/5$ & $b = 1$ satisfies (1) also

so $a = 2/5$ & $b = 1$

(83) System of equation is three equation of line in 2D system is consistent $\Rightarrow$ lines are concurrent.

$\begin{vmatrix} a & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{vmatrix} = 0$

$a(10-9) - 1(5-6) + 1(3-4) = 0$

$a = 0$

(84) $B = \begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$= \begin{vmatrix} 1 & yz & y+z \\ 0 & z(x-y) & x-y \\ 0 & y\cdot(x-z) & x-z \end{vmatrix} = (x-y)(x-z)\begin{vmatrix} 1 & yz & y+z \\ 0 & z & 1 \\ 0 & y & 1 \end{vmatrix}$

$= (x-y)(x-z)(z-y)$

$= (x-y)(y-z)(z-x)$

Similarly solve $A = (x-y)(y-z)(z-x)$

so $A = B$

(85) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3 = -(a^3+b^3+c^3-3abc)$

$= \cancel{-(a+b+c)[a^2+b^2}$

$= -(a+b+c)[a^2+b^2+c^2-ab-bc-ca]$

$= -(a+b+c)\left[\frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}\right]$

$= -$ ive

(86) $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} = 1(3a-25) - 2(a-10) + 3(5-6)$

$= a-8$

if $a-8 \neq 0 \Rightarrow$ system have unique sol.

For $a=8$ eqn are

$x_1 + 2x_2 + 3x_3 = 6$

$x_1 + 3x_2 + 5x_3 = 9$

$2x_1 + 5x_2 + 8x_3 = b$

~~Since~~ Here $2x_1 + 5x_2 + 8x_3 = (x_1 + 2x_2 + 3x_2) + (x_1 + 3x_2 + 5x_3)$

For infinite sol. $b = 6 + 9 \Rightarrow b = 15$

so $a = 8$ & $b = 15$

(87) ~~R_1~~ $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & 1+b^2x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Since $a^2+b^2+c^2 = -2$

$$= \begin{vmatrix} 1+2x+(-2x) & (1+b^2)x & (1+c^2)x \\ 1+2x+(-2x) & 1+b^2x & (1+c^2)x \\ 1+2x+(-2x) & (1+b^2)x & 1+c^2x \end{vmatrix}$$

now $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2 \Rightarrow \text{second degree}$$

(88) By symmetry B & C have equal no. of elements because by interchanging two rows or colum sign of determinant changes so (b) is correct.

Since there are determinants whose value is zero as well as ± 2 so (c) is incorrect.

(89) $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ ~~divid~~

Dividing by $p-a$, $q-b$, $r-c$

$$\Rightarrow \frac{1}{(p-a)(q-b)(r-c)} \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ \frac{a}{p-a} & \frac{q}{q-b} & \frac{c}{r-c} \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

Since $\frac{a}{p-a} = \frac{a-p+p}{p-a}$

$= -1 + \frac{p}{p-a}$

$$= \begin{vmatrix} \frac{p}{p-a} & -1+\frac{q}{q-b} & -1+\frac{r}{r-c} \\ -1+\frac{p}{p-a} & \frac{q}{q-b} & -1+\frac{r}{r-c} \\ -1+\frac{p}{p-a} & -1+\frac{q}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$ & taking common

$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2$ common from C_1

$$= \left(\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2\right) \begin{vmatrix} 1 & -1+\frac{q}{q-b} & -1+\frac{r}{r-c} \\ 1 & \frac{q}{q-b} & -1+\frac{r}{r-c} \\ 1 & -1+\frac{q}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= \left(\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2\right) \begin{vmatrix} 1 & -1+\frac{q}{q-b} & -1+\frac{r}{r-c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \left(\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2\right) \times 1 = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2 = 0$$

(90) $U_1 = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $U_2 = A^{-1}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$|A| = 1$

$C_{11} = 1$, $C_{12} = -2$, $C_{13} = 1$

$C_{21} = 0$, $C_{22} = 1$, $C_{23} = -2$ so $A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}^T$

$C_{31} = 0$, $C_{32} = 0$, $C_{33} = 1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$