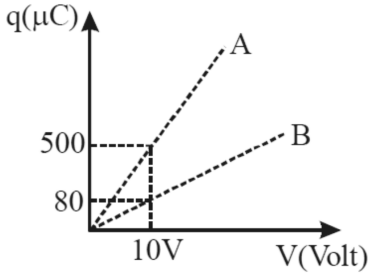
JEE-Main Exam April, 2019 / 10-4-19 / Morning session

Physics

1. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



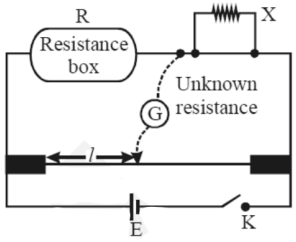
(a) 50 μF and 30 μF (b) 20 μF and 30 μF

(c) 60 μF and 40 μF (d) 40 μF and 10 μF

2. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ω m) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

(a) 1.3 m^2/Vs (b) 1.5 m^2/Vs (c) 1.8 m^2/Vs (d) 1.0 m^2/Vs

3. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure

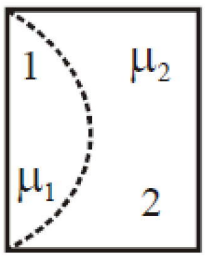


Sl. No.	R(Ω)	l(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

(a) 4 (b) 1 (c) 2 (d) 3

4. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(a) $\frac{R}{2-(\mu_1-\mu_2)}$ (b) $\frac{2R}{\mu_1-\mu_2}$

(c) $\frac{R}{2(\mu_1-\mu_2)}$ (d) $\frac{R}{\mu_1-\mu_2}$

5. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma\upsilon^2$ (where m is mass of the ball, υ is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

(a) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (b) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(c) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$ (d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

6. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that $R = 8.31\ J\ mol^{-1}\ K^{-1}$]

(a) 748 J (b) 374 J (c) 350 J (d) 700 J

7. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(a) $\frac{MR^2}{6}$ (b) $\frac{MR^2}{3}$ (c) $\frac{2MR^2}{3}$ (d) $\frac{MR^2}{2}$

8. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is

(a) $\frac{I_1\omega_1^2}{12}$ (b) $\frac{3}{8}I_1\omega_1^2$ (c) $\frac{I_1\omega_1^2}{6}$ (d) $\frac{I_1\omega_1^2}{24}$

9. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is :

(a) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$ (b) $+my_0 a\omega_1^2\hat{k}$

(c) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$ (d) Zero

10. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(a) $r_e > r_p > r_{He}$ (b) $r_e < r_p < r_{He}$

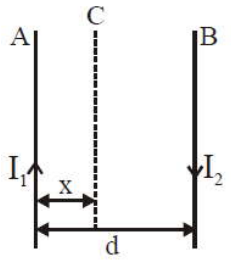
(c) $r_e, r_p = r_{He}$ (d) $r_e > r_p = r_{He}$

11. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\hat{i}\cos(kz)\cos(\omega t)$. The corresponding magnetic field $\vec{B}$ is then given by :

(a) $\vec{B} = \frac{E_0}{C}\hat{j}\sin(kz)\cos(\omega t)$ (b) $\vec{B} = \frac{E_0}{C}\hat{j}\sin(kz)\sin(\omega t)$

(c) $\vec{B} = \frac{E_0}{C}\hat{k}\sin(kz)\cos(\omega t)$ (d) $\vec{B} = \frac{E_0}{C}\hat{j}\cos(kz)\sin(\omega t)$

12. Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are:



(a) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

(b) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

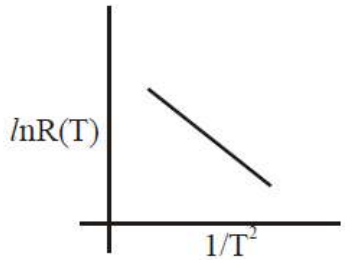
(c) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \frac{I_2}{(I_1 - I_2)}d$

(d) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

13. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

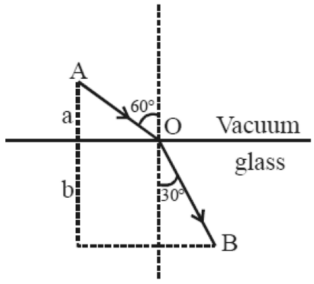
(a) 4; 1×10^8 Hz (b) 0.25; 1×10^8 Hz (c) 4; 2×10^8 Hz (d) 0.25; 2×10^8 Hz

14. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that :



(a) $R(T) = \frac{R_0}{T^2}$ (b) $R(T) = R_0 e^{-T^2/T_0^2}$ (c) $R(T) = R_0 e^{-T_0^2/T^2}$ (d) $R(T) = R_0 e^{T^2/T_0^2}$

15. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :



(a) 2a + 2b (b) $2a + \frac{2b}{3}$ (c) $\frac{2\sqrt{3}}{a} + 2b$ (d) $2a + \frac{2b}{\sqrt{3}}$

16. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are :

(a) 220 V and 10 A (b) 440 V and 5 A (c) 440 V and 20 A (d) 220 V and 20 A

17. In a photoelectric effect experiment the threshold wavelength of the light is 380 nm. If the wavelentgh of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be:

Given E (in eV) $= \dfrac{1237}{\lambda(\text{in nm})}$

(a) 1.5 eV (b) 4.5 eV (c) 15.1 eV (d) 3.0 eV

18. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-01.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to:

(a) 13 s (b) 7 s (c) 27 s (d) 4 s

19. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. There fore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is:

(a) 200 Ω (b) 100 Ω (c) 10 Ω (d) 500 Ω

20. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s)

(a) 16, 14 (b) 12, 18 (c) 12, 16 (d) 8, 18

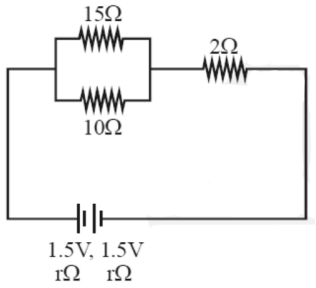
21. The radioactive materials A and B have decay constants 10λ and λ, respectively. It initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time :

(a) $\dfrac{11}{10\lambda}$ (b) $\dfrac{1}{9\lambda}$ (c) $\dfrac{1}{10\lambda}$ (d) $\dfrac{1}{11\lambda}$

22. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is :

(a) 60 (b) 10^4 (c) 6×10^2 (d) 10^2

23. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2V. The internal resistance r, of each cell is :



(a) 1 Ω (b) 1.5 Ω (c) 0 Ω (d) 0.5 Ω

24. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?

(a) $\sim 10^{11}$ (b) $\sim 10^{13}$ (c) $\sim 10^{10}$ (d) $\sim 10^{12}$

25. n moles of an ideal gas with constant volume heat capacity C_v undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(a) $\dfrac{4nR}{C_v - nR}$ (b) $\dfrac{nR}{C_v - nR}$ (c) $\dfrac{nR}{C_v + nR}$ (d) $\dfrac{4nR}{C_v + nR}$

26. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed *u* at z = 4a. The minimum value of *u* such that it crosses the origin is :

(a) $\sqrt{\dfrac{2}{m}\left(\dfrac{1}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)}^{1/2}$ (b) $\sqrt{\dfrac{2}{m}\left(\dfrac{2}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)}^{1/2}$ (c) $\sqrt{\dfrac{2}{m}\left(\dfrac{4}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)}^{1/2}$ (d) $\sqrt{\dfrac{2}{m}\left(\dfrac{1}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)}^{1/2}$

27. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m)
(a) 1.6×10^6 m (b) 6.4×10^6 m (c) 9.0×10^6 m (d) 2.6×10^6 m

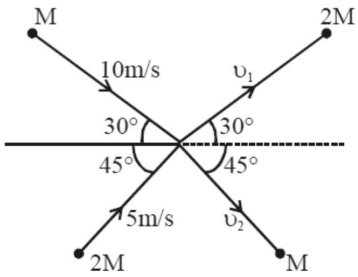
28. Given below in the left column are different modes of communication using the kinds of waves given the right column.

A.	Optical Fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

(a) A-S, B-Q, C-R, D-P (b) A-R, B-P, C-S, D-Q
(c) A-Q, B-S, C-R, D-P (d) A-Q, B-S, C-P, D-R

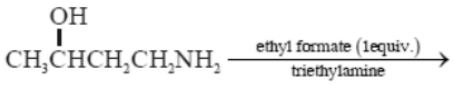
29. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :
(a) 2/3 (b) 3/5 (c) 2/5 (d) 4/5

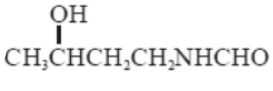
30. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds υ_1 and υ_2, respectively. The values of υ_1 and υ_2 are nearly :
(a) 3.2 m/s and 6.3 m/s
(b) 3.2 m/s and 12.6 m/s
(c) 6.5 m/s and 6.3 m/s
(d) 6.5 m/s and 3.2 m/s

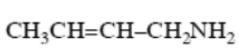


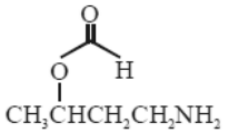
Chemistry

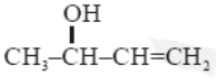
31. The major product of the following reaction is :



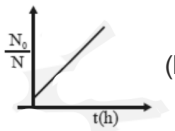
(a) 

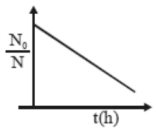
(b) 

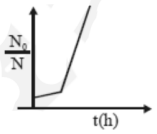
(c) 

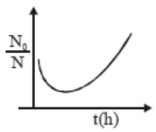
(d) 

32. A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as π. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour?

(a) 

(b) 

(c) 

(d) 

33. The correct order of catenation is :
(a) C > Si > Ge ≈ Sn (b) C > Sn > Si ≈ Ge
(c) Ge > Sn > Si > C (d) Si > Sn > C > Ge

34. The oxoacid of sulphur that does not contain bond between sulphur atoms is :
(a) $H_2S_4O_6$ (b) $H_2S_2O_7$ (c) $H_2S_2O_3$ (d) $H_2S_2O_4$

35. Consider the statements S1 and S2 : S1 : Conductivity always increases with decrease in the concentration of electrolyte. S2 : Molar conductivity always increases with decrease in the concentration of electrolyte. The correct option among the following is :
(a) Both S1 and S2 are correct (b) S1 is wrong and S2 is correct
(c) S1 is correct and S2 is wrong (d) Both S1 and S2 are wrong

36. Which of the following is a condensation polymer?
(a) Buna - S (b) Nylon 6, 6 (c) Teflon (d) Neoprene

37. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :
(a) C_4H_8 (b) C_4H_7Cl (c) C_4H_{10} (d) C_4H_6

38. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with :
(a) $NaBH_4$ (b) CaH_2 (c) H_2O (d) NH_2NH_2

39. The isoelectronic set of ions is :
(a) N^{3-}, Li^+, Mg^{2+} and O^{2-} (b) Li^+, Na^+, O^{2-} and F^-
(c) F^-, Li^+, Na^+ and Mg^{2+} (d) N^{3-}, O^{2-}, F^- and Na^+

40. The species that can have a trans-isomer is : (en = ethane-1, 2-diamine, ox = oxalate)
(a) $[Pt(en)Cl_2]$ (b) $[Cr(en)_2(ox)]^+$ (c) $[Zn(en)Cl_2]$ (d) $[Pt(en)_2Cl_2]^{2+}$

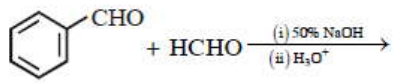
41. Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)	Column II (Metals)
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

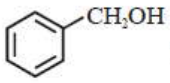
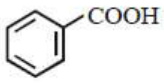
(a) (I) – (B); (II) – (C); (III) – (D); (IV) – (A)
(b) (I) – (B); (II) – (D); (III) – (A); (IV) – (C)
(c) (I) – (C); (II) – (A); (III) – (B); (IV) – (D)
(d) (I) – (C); (II) – (D); (III) – (B); (IV) – (A)

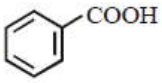
42. Consider the following statements
(i) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.
(ii) Ionic product of water is temperature dependent.
(iii) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.
(iv) The Le Chatelier's principle is not applicable to common-ion effect.
the correct statement are :
(a) (i), (ii) and (iv) (b) (i), (ii) and (iii) (c) (i) and (ii) (d) (ii) and (iii)

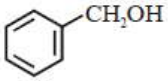
43. Major products of the following reaction are :



(a) CH_3OH and HCO_2H

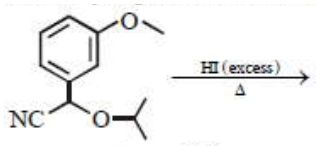
(b)  and 

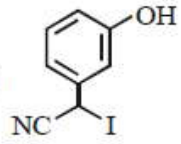
(c) CH_3OH and 

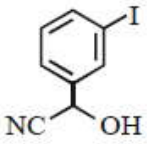
(d) HCOOH and 

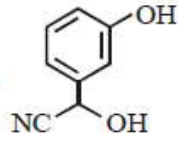
44. The principle of column chromatography is :
(a) Capillary action.
(b) Gravitational force.
(c) Differential adsorption of the substances on the solid phase.
(d) Differential absorption of the substances on the solid phase.

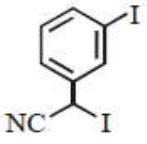
45. The major product of the following reaction is :



(a) 

(b) 

(c) 

(d) 

46. Amylopectin is composed of :
(a) α-D-glucose, C_1–C_4 and C_1–C_6 linkages (b) α -D-glucose, C_1–C_4 and C_2–C_6 linkages
(c) β -D-glucose, C_1–C_4 and C_2–C_6 linkages (d) β -D-Glucose, C_1–C_4 and C_1–C_6 linkages

47. Consider the hydrates ions of Ti^{2+}, V^{2+}, Ti^{3+} and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(a) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$ (b) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$

(c) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$ (d) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

48. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increases with :

(a) Decrease in p and decrease in T (b) Increase in p and increase in T

(c) Increase in p and decrease in T (d) Decrease in p and increase in T

49. Three complexes,

$[CoCl(NH_3)_5]^{2+}$ (I),

$[Co(NH_3)_5H_2O]^{3+}$ (II) and

$[Co(NH_3)_6]^{3+}$ (III)

absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

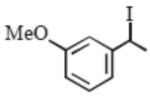
(a) (III) > (I) > (II) (b) (I) > (II) > (III) (c) (II) > (I) > (III) (d) (III) > (II) > (I)

50. During the change of O_2 to O_2^- the incoming electron goes to the orbital :

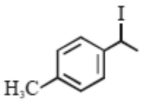
(a) $\sigma^{*2}p_z$ (b) $\pi^2 p_y$ (c) $\pi^{*2}p_x$ (d) $\pi^2 p_x$

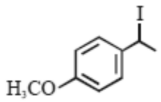
51. Increasing rate of S_N1 reaction in the following compounds is :


(A)



(B)





(a) (A) < (B) < (C) < (D) (b) (B) < (A) < (D) < (C)

(c) (B) < (A) < (C) < (D) (d) (A) < (B) < (D) < (C)

52. Consider the following table :

Gas	a/(k Pa dm^6 mol^{-1})	b/(dm^3 mol^{-1})
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are vander waals constant. The correct statement about the gases is :

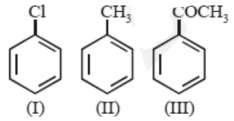
(a) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(b) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

(c) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

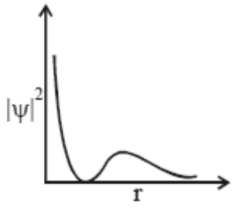
(d) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

53. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :-



(a) I < III < II (b) II < I < III (c) III < I < II (d) III < II < I

54. The graph between $|\psi|^2$ and r(radial distance) is shown below. This represents :-



(a) 3s orbital (b) 1s orbital (c) 2p orbital (d) 2s orbital

55. At room temperature, a dilute solution of urea is prepared by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be (molar mass of urea = 60 g mol^{-1}):-

(a) 0.027 mmHg (b) 0.028 mmHg (c) 0.017 mmHg (d) 0.031 mmHg

56. The synonym for water gas when used in the production of methanol is :-

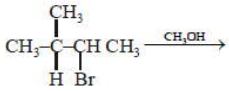
(a) natural gas (b) laughing gas (c) syn gas (d) fuel gas

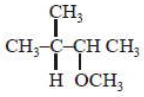
57. A process will be spontaneous at all temperatures if :-

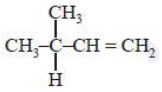
(a) $\Delta H > 0$ and $\Delta S < 0$ (b) $\Delta H < 0$ and $\Delta S > 0$

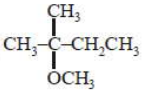
(c) $\Delta H > 0$ and $\Delta S > 0$ (d) $\Delta H < 0$ and $\Delta S < 0$

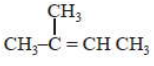
58. The major product of the following reaction is :-



(a) 

(b) 

(c) 

(d) 

59. The regions of the atmosphere, where clouds form and where we line respectively, are :-

(a) Stratosphere and Troposphere (b) Troposphere and Stratosphere

(c) Troposphere and Troposphere (d) Stratosphere and Stratosphere

60. The alloy used in the construction of aircrafts is :-

(a) Mg – Sn (b) Mg – Mn (c) Mg – Al (d) Mg – Zn

Mathematics

61. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(a) 2.8 (b) 3.2 (c) 3.0 (d) 2.5

62. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(a) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$ (b) $\Delta_1 + \Delta_2 = -2x^3$

(c) $\Delta_1 - \Delta_2 = -2x^3$ (d) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

63. $\lim\limits_{x\to 1}\frac{x^4-1}{x-1} = \lim\limits_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is :

(a) $\frac{3}{8}$ (b) $\frac{3}{2}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

64. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(a) 12 (b) 10 (c) 9 (d) 7

65. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(a) exactly two values of K (b) exactly one value of K

(c) no value of K. (d) infinitely many values of K

66. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true?

(a) $f(g(S)) \neq f(S)$ (b) $f(g(S)) = S$ (c) $g(f(S)) = g(S)$ (d) $g(f(S)) \neq S$

67. Let $f(x) = e^x - x$ and $g(x)$ $x^2 - x, \forall x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is:

(a) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (b) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$ (c) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (d) $[0, \infty)$

68. Which one of the following Boolean expressions is a tautology?

(a) $(p \vee q) \wedge (\sim p \vee \sim q)$ (b) $(p \wedge q) \vee (p \wedge \sim q)$

(c) $(p \vee q) \wedge (p \vee \sim q)$ (d) $(P \vee q) \vee (p \vee \sim q)$

69. All the pairs (x, y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4\sin^2 y} \leq 1$ also satisfy the equation.

(a) $\sin x = |\sin y|$ (b) $\sin x = 2 \sin y$ (c) $2|\sin x| = 3 \sin y$ (d) $2 \sin x = \sin y$

70. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(a) 36 (b) 60 (c) 48 (d) 72

71. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(a) $\frac{1}{11}$ (b) $\frac{1}{17}$ (c) $\frac{1}{10}$ (d) $\frac{1}{12}$

72. The sum $\frac{3\times 1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7x(1^3+2^3+3^3)}{1^2+2^2+3^2}+\ldots$

(a) 660 (b) 620 (c) 680 (d) 600

73. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(a) $4e^4 - 24e^2 + 35 = 0$ (b) $4e^4 + 8e^2 - 35 = 0$

(c) $4e^4 - 12e^2 - 27 = 0$ (d) $4e^4 - 24e^2 + 27 = 0$

74. If $f(x) = \begin{cases} \frac{\sin(p+1)+\sin x}{x} & , \ x<0 \\ q & , \ x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , \ x>0 \end{cases}$

is continuous at $x = 0$, then the ordered pair (p, q) is equal to:

(a) $\left(\frac{5}{2},\frac{1}{2}\right)$ (b) $\left(-\frac{3}{2},-\frac{1}{2}\right)$ (c) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (d) $\left(-\frac{3}{2},\frac{1}{2}\right)$

75. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x, x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(a) $2+\frac{1}{e}$ (b) $\frac{1}{2}-e$ (c) $e - 2$ (d) $\frac{1}{2}-e$

76. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus recturm of the ellipse is

(a) 9 (b) $8\sqrt{3}$ (c) $12\sqrt{2}$: (d) 5

77. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x]\,dx$, where [t] denotes the greatest integer function, is:

(a) -2π (b) π (c) $-\pi$ (d) 2π

78. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a:

(a) square of side length $2\sqrt{2}$ units (b) rhombus of side length 2 units

(c) square of area 16 sq, units (d) rhombus of area $8\sqrt{2}$ sq. units.

79. The line $x = y$ touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

(a) $3\sqrt{2}$ (b) 3 (c) $2\sqrt{2}$ (d) 2

80. Let A(3, 0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2:1, then cos ($\angle$GOA) (O being the origin) is equal to:

(a) $\frac{1}{\sqrt{30}}$ (b) $\frac{1}{6\sqrt{10}}$ (c) $\frac{1}{\sqrt{15}}$ (d) $\frac{1}{2\sqrt{15}}$

81. Let $f: R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is:

(a) differentiable if f' (c) = 0 (b) not differentiable

(c) differentiable if f' (c) $\neq$ 0 (d) not differentiable if f' (c) = 0

82. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to :

(a) $\frac{2^6}{(\sin\theta + 8)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta - 8)^6}$ (c) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (d) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$

83. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(a) -1 (b) 2 (c) -2 (d) 1

84. If $\int \frac{dx}{(x^2 - 2x + 10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2 - 2x + 10}\right) + C$ where C is a constant of integration, then :

(a) $A = \frac{1}{27}$ and $f(x) = 9(x - 1)$ (b) $A = \frac{1}{81}$ and $f(x) = 3(x - 1)$

(c) $A = \frac{1}{54}$ and $f(x) = 9(x - 1)^2$ (d) $A = \frac{1}{54}$ and $f(x) = 3(x - 1)$

85. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\text{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is:

(a) $10\sqrt{5}$ (b) $\frac{100}{3\sqrt{3}}$ (c) 20 (d) 25

86. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(a) 38 (b) 98 (c) 76 (d) 64

87. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots + \frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(a) $\frac{4}{3}(2)^{4/3}$ (b) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$ (c) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$ (d) $\frac{4}{3}(2)^{3/4}$

88. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(a) $\frac{\sqrt{65}}{2}$ (b) $\frac{\sqrt{91}}{4}$ (c) $2\sqrt{13}$ (d) $\frac{\sqrt{91}}{2}$

89. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(a) (28, 315) (b) (–54, 315) (c) (–21, 714) (d) (24, 861)

90. If $a > 0$ and $z = \frac{(1+i)^2}{a - i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to:

(a) $-\frac{3}{5} - \frac{1}{5}i$ (b) $-\frac{1}{5} + \frac{3}{5}i$ (c) $-\frac{1}{5} - \frac{3}{5}i$ (d) $\frac{1}{5} - \frac{3}{5}i$

PCM Answers

1	d	11	b	21	b	31	a	41	d	51	c	61	a	71	a	81	a
2	d	12	b	22	d	32	a	42	b	52	c	62	b	72	a	82	d
3	a	13	d	23	d	33	a	43	d	53	c	63	d	73	a	83	a
4	d	14	c	24	d	34	b	44	c	54	d	64	b	74	d	84	d
5	b	15	a	25	c	35	b	45	c	55	c	65	c	75	c	85	c
6	a	16	b	26	b	36	b	46	a	56	c	66	c	76	a	86	c
7	c	17	a	27	d	37	d	47	a	57	b	67	b	77	c	87	c
8	d	18	b	28	d	38	d	48	c	58	c	68	d	78	a	88	d
9	b	19	c	29	c	39	d	49	b	59	c	69	a	79	a	89	a
10	c	20	b	30	c	40	d	50	c	60	c	70	b	80	c	90	c